Predicting the modulus of elasticity of clayey soil composites reinforced with scrap tire rubber fibers using a composite material model

Previsão do módulo de elasticidade de compósitos de solo argiloso reforçados com fibras de borracha de pneus usados utilizando um modelo de material compósito

Predicción del módulo de elasticidad de materiales compuestos de suelos arcillosos reforzados con fibras de caucho de neumáticos usados mediante un modelo de material compuesto

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ABSTRACT
In this study, composite material models are used to predict the modulus of elasticity of a composite consisting of scrap tire rubber fibers and clayey soils. The calculated modulus of elasticity is compared with the reference modulus obtained from experimental testing. Predicting the effective mechanical properties of composites is crucial in situations where testing is impractical, challenging, or costly. The analysis involves various approaches within the elasticity framework, utilizing rheological models such as Voigt, Reuss, Hirsch-Dougill, Popovics, Halpin-Tsai, Hashin, and the Bache & Napper–Christensen estimation. These models aim to predict the effective Young's modulus of the composite system comprising soil and rubber fibers. The maximum discrepancies observed are 10.66%, 12.71%, and 12.98% for both soils. Voigt, Hashin, and Bache estimations provide highly accurate predictions of the effective Young's modulus, showing excellent agreement with experimental results across different fiber volume fractions ranging from 10% to 50%.

Keywords: Laws of Mixture. Scrap Tyre Rubber Fibre. Effective Young’s Modulus. Composite Materials. Soil Mixtures.

RESUMO
Neste estudo, modelos de materiais compostos são usados para prever o módulo de elasticidade de um composto que consiste em fibras de borracha de pneu de
sucata e solos argilosos. O módulo de elasticidade calculado é comparado com o módulo de referência obtido de testes experimentais. Prever as propriedades mecânicas efetivas de compostos é crucial em situações em que os testes são impraticáveis, desafiadores ou caros. A análise envolve várias abordagens dentro da estrutura de elasticidade, utilizando modelos reológicos como Voigt, Reuss, Hirsch-Dougill, Popovics, Halpin-Tsai, Hashin e a estimativa de Bache & Napper–Christensen. Esses modelos visam prever o módulo de Young efetivo do sistema composto que compreende solo e fibras de borracha. As discrepâncias máximas observadas são 10,66%, 12,71% e 12,98% para ambos os solos. As estimativas de Voigt, Hashin e Bache fornecem previsões altamente precisas do módulo de Young efetivo, mostrando excelente concordância com resultados experimentais em diferentes frações de volume de fibra variando de 10% a 50%.


1 INTRODUCTION

Used tires are abundant and concerning waste. Aggregates derived from grinding used tires are increasingly used in the field of civil engineering (geotechnical, hydraulic structures, lightweight concrete, asphalt concrete, etc.). Depending on the type of used tires, dimensions, and any separations and treatments, the physical and mechanical properties of these fibers may change.
This type of material, such as fiber concrete, fiber plaster, or fiber-reinforced soils, as well as certain natural materials like bone tissues, presents a microstructure consisting of a matrix and a distribution of fibers oriented continuously in all directions (randomly) Fritsch.2009. Evaluating the effective behavior of composite using analytical methods requires a profound understanding of the various approaches existing in the abundant literature in this field. As explained by Gilormini; Bréchet (1999), the choice of a model is governed by several parameters, including the geometry of the heterogeneous medium, the mechanical contrast between phases, and the volume fraction of reinforcements. Some researchers have used models to estimate composites, and others have developed different types of models to predict the elastic properties of specific types such as dam concrete (Topçu, 2005; Vilardeil et al., 1998), rubberized concrete (Topçu, 1997; Topçu; Avcular, 1994; Topçu, 1994), silica fume concrete (Mostofinejad; Nozhati. 2005), and concretes produced with different types of aggregates (Simeonov; Ahmad, 1995; Nilsen; Monterio, 1993). The prediction of effective mechanical properties of composites using appropriately validated mixing laws is of great practical interest in all circumstances where tests are impossible, difficult, or expensive.

Our study revolves around determining the effective Young's modulus of soil (matrix) reinforced with fibers from recycled rubber tires, comparing it to the effective Young's modulus of experimentally tested soil-fiber composites.

2 MATERIALS USED

2.1 THE MATRIX PHASE (CLAY) AND ELASTIC YOUNG'S MODULUS

Two soils of different origin and physical properties were chosen. The first sample comes from the Ayaida (A) region of Oran in northwest Algeria. This soil was the cause of several disorders in the buildings. The second sample is Bentonite (B) supplied by the Bental unit of Maghnia in northwest Algeria. Figure 1 shows the molecular size distribution of the two soils.

Young's modulus (e), commonly referred to as the elastic modulus of soil, is a soil elastic parameter and a measure of soil stiffness. It is defined as the ratio
of the stress along an axis above the stress along that axis in the range of elastic behavior of soil. Elastic modulus is often used for the evaluation of ground support and elastic deformation analysis. The elastic modulus of soil can be estimated from laboratory or in-situ tests or be based on correlation with other soil properties. In the laboratory, it can be determined from the three-axis test or indirectly from the odometer test. On the field, it can estimate from standard penetration test, cone penetration test, pressuremeter or indirectly from dilatometer test.

Typical Values of Young's Modulus for Cohesive Material (MPA) (based on Obrzud; Truty. 2012 compiled from Kezdi. 1974; Prat et al. 1995).

According to the Unified Soil Classification System (USCS), Ayaida soil is defined as clay with low plasticity (CL). The main constituent of Ayaida soil is silica. And Maghnia Bentonite is defined as high-plasticity clay (CH). Young's modulus of Ayaida soil is 3.509MPa, and Young's modulus of Maghnia Bentonite is 3.972MPa.
2.2 THE REINFORCEMENT PHASE (RUBBER FIBERS) AND ELASTIC YOUNG'S MODULUS

The rubber fibers used in this work come from the processing of used tires by crushing. The steel fibers are separated from the powder magnetically. Figure 2 Photograph showing scrap tyre rubber fibre.

In addition to fine rubber particles, the powder contains textile fibers. The principle of this experimental campaign is to make test pieces of composite soils and their mixtures with different identical fiber contents (10%, 20%, 25% and 50%) of the same water content and density (homogeneous and isotropic material). The materials are packaged in hermetically sealed bags and are stored at the ambient temperature of the test room (~20°C). The rest time necessary to ensure homogeneous distribution of water within the sample for all composite materials.

The study of Young's modulus of rubber tire itself has been studied by many authors. A study of tire chips from three different suppliers, using vertical and horizontal stress compressibility measurements for loading/unloading at low stress levels (Humphrey et al., 1993).

The values of Young's modulus vary from 1.2 to 5.1 MPa and the mean values of the Poisson ratio range 0.20-0.32. The triaxial chip test of tires from 0.08 to 2.01 in (2 to 51 mm) in size (Bressette, 1984; Ahmed, 1993; Masad et al., 1996; Wu et al., 1997).

Tests were carried out in compression loading by Wu et al. (1997), in addition, the loading-unloading test was carried out where the confining pressure $\sigma_3$ was reduced in steps from the initial consolidation pressure while increasing the vertical load to keep it constant $\sigma_1$. The initial tangent modulus of the stress-strain curves, analogous to Young's modulus, ranges between 0.3 and 2.5 MPa, with higher values at higher confinement stresses. Hernández-Olivares and Barluenga (2004) present nominal rubber properties of truck tires, Young's modulus ranges from 1.97 MPa to 22.36 MPa, $E = 1.97$ MPa is evaluated at 100% strain. The average value of Young's modulus of waste tire rubber $E_A$ is equal to 2.214 MPa which is found in my research. The Poisson's ratio of scrap tire rubber is assumed to be 0.45.
3 EXPERIMENTAL YOUNG’S MODULUS OF THE TWO SOILS STUDIED

The test was carried out according to the standard (ASTM D 2435) on all compacted samples. This test was carried out in the conventional Oedometer 50 mm in diameter and 20 mm thick. The load is applied in stages kept constant, successively increasing according to a defined program (0; 0.013; 0.025; 0.051; 0.102; 0.408; 0.815; 1.630) MPa. Variations in the height of the specimen are measured during the test as a function of the duration of application of the load. The principle of this experimental campaign is to make test pieces of composite soils and their mixtures with different fiber contents (0%, 10%, 20%, 25%, and 50%) identical with the same water content and density (homogeneous material and isotropic). The materials are packaged in hermetically sealed bags and are stored at the ambient temperature of the test room (~20°C). The rest time is necessary to ensure a homogeneous distribution of water within the sample for all composite materials. The composite soil is placed in a rigid envelope, a variable pressure is applied using a piston, and the subsidence observed after stabilization is measured, ASTM D 2435.

The oedometric module noted $E'$ varies according to the pressures corresponding to the stress interval (Sanglerat).
\[
\Delta \sigma = \sigma_i - \sigma, \quad E' = \frac{1 + e}{c_c} \frac{\Delta \sigma}{\log (1 + \Delta \sigma / \sigma)} \quad (1)
\]

With \( e \) the vacuum index and \( \sigma \) the stress characterize the initial state.

\[
E = E' \left(1 - \frac{2\nu^2}{\nu}\right) \quad (2)
\]

where:

\( \nu \) is the Poisson's ratio of the materials.

We can admit that this formula applies to soils as a first approximation. The Poisson's ratio of soils is poorly known and few determinations have been made. However, it seems that \( \nu = 0.33 \) constitutes a reasonable estimate. We will therefore write, adopting this value:

\[
E = \frac{2}{3} E' \quad (3)
\]

Table (1) compiles the Young's modulus of experimentally measured soil-fiber composite \( E_c^{\text{exp}} \).

These first tests make it possible to trace the reference curves to then compare them with tests on soil–rubber fiber composites with different volume fractions.

Bibliographic data for different experimentally tested soils are compiled in Table 2. The mechanical data are \( E_m \) the Young's modulus of the matrix, \( E_a \) that of the rubber fibers and \( E_c^{\text{exp}} \) the Young's modulus of the soil measured experimentally.

As well as different volume fractions for soil-fiber composites. In our work we tested two types of clay soils with Young's modulus (3.509 and 3.972) MPa. Each type of matrix soil is mixed with reinforcement of rubber fibers having Young's modulus (2.214) MPa. Which corresponds to a contrast ratio between the two phases varying from \( (E_a/E_m) \) varying from 0.56 to 0.63.
3 CHOICE OF ANALYTICAL MODELS

As explained by Gilormini and Bréchet (1999), the choice of a model is governed by several parameters, and in particular by the geometry of the heterogeneous medium, the mechanical contrast between the matrixes phases (soils) and the volume fraction of the reinforcements (rubber fibers).

Remember that in our case, we are studying a heterogeneous composite material (soil-rubber fibers from worn tires) composed of two homogeneous phases: the fibers (or the inclusion; phase a) of volume fraction Va, embedded in the soil (or matrix, phase m). Each of these two phases presents a linear, homogeneous and isotropic elastic behavior. The two phases are assumed to be perfectly stuck together (matrix/inclusion bond, perfect adhesion is assumed), which justifies the biphasic representation. Used tire rubber fibers are evenly distributed in the soil matrix. The multi-phase description of composite materials made up of an elastic matrix. The determination of the effective properties then consists in this case of defining the homogeneous equivalent soil-fiber behavior
based on the characteristics of the soil and the rubber fibers of the worn tires. The two mixing laws of Voigt and Reuss are simple and incapable of determining with acceptable precision the modulus of elasticity of the composite because they do not take into account the morphology of the material, the discontinuity of the reinforcement, nor its orientation and of the true nature of matrix/reinforcement interface, these two terminals are enriched by combining them with each other. Two combinations proposed by Larrard (1995). Le module d'élasticité effectif est donné pour les deux modèles (Voigt et Reuss) par les équations (4) et (5). The biphasic models of Hirsch-Dougill, Popovics, Halpin-Tsai, Hashin and Bache and Napper-Christensen originally designed for particle composites Nielsen et el., 1968, propose effective elastic modulus of the composite by equations (6), (7), (8), (9) and (10) successively.

\[ E_{CVoigt} = E_m V_m + E_a V_a \]  \hspace{1cm} (4)

\[ E_{CReuss} = \frac{E_m}{V_m} + \frac{E_a}{V_a} \]  \hspace{1cm} (5)

\[ E_{CHirsh-Dougill} = \frac{1}{2} \left( \frac{1}{E_{CVoigt}} + \frac{1}{E_{CReuss}} \right) \]  \hspace{1cm} (6)

\[ E_{CPopovics} = \frac{1}{2} \left( E_{CVoigt} + E_{CReuss} \right) \]  \hspace{1cm} (7)

\[ E_{CHalpin-Tsai} = \frac{3}{8} E_{CVoigt} + \frac{5}{8} E_{CReuss} \]  \hspace{1cm} (8)

\[ E_{CHalpin} = \frac{(E_a + E_m) + (E_a - E_m)V_a}{(E_a + E_m) - (E_a - E_m)V_a} E_m \]  \hspace{1cm} (9)

\[ E_{CHalpin} = E_m^V_a E_a^V \]  \hspace{1cm} (10)
where:

- $E_m$ is the modulus of elasticity of the soil matrix,
- $E_a$ is the modulus of elasticity of the fiber phase,
- $E_C$ is the modulus of elasticity of soil-fibers,
- $V_m$ is the volume fraction of the soil as matrix and
- $V_a$ is the volume fraction of the rubber fibers of the reinforcement phase.

$E_C$ Takes the indices of the composite model used for estimation of the elastic modulus. For example, $E_C$ Voigt expresses the elastic modulus of soil-fibers estimated using the Voigt composite model.

4 RESULTS OF PREDICTIVE APPROACHES AND DISCUSSION

4.1 EFFECT OF FIBER CONTENT ON THE DRY DENSITY OF TWO CLAY SOILS

Based on the results depicted in Figure 3, it is evident that the dry density of fiber-reinforced soils gradually decreases with the increasing content of fibers for both soil types A and B. This reduction is attributed to the decrease in the average unit weight of solids within the mixture. The test results indicate that the inclusion of fibers leads to a decrease in the maximum dry density in both soils A and B, primarily due to the lower density of the added fibers. The decline in the mixture’s density can be attributed to several factors. Firstly, the rubber fibers have a smaller specific gravity compared to the soil. In this study, the dry density of clay B and A, and rubber fibers were 1.98 g/m$^3$, 1.93 g/m$^3$, and 1.20 g/cm$^3$, respectively. Secondly, the flexibility of rubber fibers contributes to enhanced compaction efficiency, resulting in a reduced maximum dry density of the soil and fiber mixture. Thirdly, the lower optimum moisture content of clay and fibers, influenced by the poor water absorption of rubber fibers compared to clay. For instance, an increase in the content of tyre rubber fibers from 10% to 50% leads to a reduction in density by 26.67% and 26.88% for soil A and B, respectively.

These findings align with previous studies (Seda et al., 2007; Dunham-Friel and Carraro, 2014) indicating that, while the optimum water content remains
unchanged, the maximum dry unit weight of the mixture decreases by about 0.3 g/cm³ with the addition of 20% rubber by weight. Bekhiti et al. (2019) also observed a decrease in dry density with an increase in waste tyre rubber fiber content. The highest dry density value, 1.29 g/cm³, was achieved with a 30% PVC waste aggregate content. Interestingly, this maximum dry density is 17.83% lower than the unreinforced samples (Karboua et al., 2023).

![Figure 3. Variation of dry density of different mixtures.](Image)

The mixtures of two clayey soils

Source: The authors.

4.2 COMPARISON OF ANALYTICAL MODELS WITH EXPERIMENTAL RESULTS

Table 3 shows our results from the application of analytical approaches and those obtained by experimental method. In order to derive the closest model to calculate the effective Young’s modulus, we calculated the differences between the different predictive models and the experimental results (see Table 4). The average relative differences between the measured experimental $E_c^{Exp}$ Young’s modulus and the numerical predictions $E_c^{Cal}$ are calculated by the relation below:

$$E_{cart} = \left| \frac{E_c^{Cal} - E_c^{Exp}}{E_c^{Exp}} \right| \times 100$$  \hspace{1cm} (11)
Table 3. Effective modulus of elasticity (MPa): Comparative analysis of various analytical models against experimental results.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>( E_{\text{CVoigt}} )</th>
<th>( E_{\text{CReuss}} )</th>
<th>( E_{\text{CHirsch}} )</th>
<th>( E_{\text{CPopovics}} )</th>
<th>( E_{\text{CHalpin-Tsai}} )</th>
<th>( E_{\text{CHashin}} )</th>
<th>( E_{\text{CBache}} )</th>
<th>( E_{\exp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>3.509</td>
<td>0.285</td>
<td>0.527</td>
<td>1.897</td>
<td>1.494</td>
<td>3.509</td>
<td>3.509</td>
<td>3.509</td>
</tr>
<tr>
<td>A2</td>
<td>3.38</td>
<td>0.302</td>
<td>0.543</td>
<td>1.841</td>
<td>1.456</td>
<td>3.534</td>
<td>3.351</td>
<td>3.885</td>
</tr>
<tr>
<td>A3</td>
<td>3.25</td>
<td>0.318</td>
<td>0.56</td>
<td>1.784</td>
<td>1.418</td>
<td>3.205</td>
<td>3.2</td>
<td>3.195</td>
</tr>
<tr>
<td>A4</td>
<td>3.185</td>
<td>0.327</td>
<td>0.569</td>
<td>1.756</td>
<td>1.399</td>
<td>3.133</td>
<td>3.127</td>
<td>3.221</td>
</tr>
<tr>
<td>A5</td>
<td>2.862</td>
<td>0.368</td>
<td>0.619</td>
<td>1.615</td>
<td>1.303</td>
<td>2.796</td>
<td>2.787</td>
<td>3.203</td>
</tr>
<tr>
<td>B1</td>
<td>3.972</td>
<td>0.252</td>
<td>0.474</td>
<td>2.112</td>
<td>1.647</td>
<td>3.972</td>
<td>3.972</td>
<td>3.972</td>
</tr>
<tr>
<td>B2</td>
<td>3.796</td>
<td>0.272</td>
<td>0.492</td>
<td>2.034</td>
<td>1.593</td>
<td>3.752</td>
<td>3.747</td>
<td>3.472</td>
</tr>
<tr>
<td>B3</td>
<td>3.62</td>
<td>0.292</td>
<td>0.511</td>
<td>1.956</td>
<td>1.54</td>
<td>3.545</td>
<td>3.534</td>
<td>3.455</td>
</tr>
<tr>
<td>B4</td>
<td>3.533</td>
<td>0.302</td>
<td>0.522</td>
<td>1.917</td>
<td>1.513</td>
<td>3.445</td>
<td>3.432</td>
<td>3.396</td>
</tr>
<tr>
<td>B5</td>
<td>3.093</td>
<td>0.352</td>
<td>0.581</td>
<td>1.722</td>
<td>1.38</td>
<td>2.984</td>
<td>2.965</td>
<td>3.316</td>
</tr>
</tbody>
</table>

Source: The authors.

Table 4. Percentage deviations in Young's modulus between analytical model predictions and experimental results.

| Ref. | \( E_{\text{CVoigt}} \) | \( E_{\text{CReuss}} \) | \( E_{\text{CHirsch}} \) | \( E_{\text{CPopovics}} \) | \( E_{\text{CHalpin-Tsai}} \) | \( E_{\text{CHashin}} \) | \( E_{\text{CBache}} \) |
|------|------------------|-----------------|-----------------|------------------|------------------|------------------|------------------|------------------|
| A1   | 0.00             | 91.88           | 84.98           | 45.94            | 57.42            | 0.00             | 0.00             | 2.88             |
| A2   | 6.11             | 90.53           | 82.95           | 42.21            | 54.29            | 5.30             | 5.12             | 9.88             |
| A3   | 1.37             | 90.34           | 83.00           | 45.85            | 56.97            | 2.73             | 2.71             | 5.68             |
| A4   | 1.11             | 89.86           | 82.33           | 45.48            | 56.58            | 2.72             | 2.72             | 5.22             |
| A5   | 10.66            | 88.50           | 80.67           | 49.58            | 59.11            | 12.72            | 12.99            | 17.88            |
| B1   | 0.00             | 93.66           | 88.08           | 46.83            | 58.54            | 0.00             | 0.00             | 2.88             |
| B2   | 9.34             | 96.17           | 85.83           | 41.42            | 54.11            | 8.08             | 7.92             | 16.68            |
| B3   | 4.79             | 91.56           | 85.21           | 43.38            | 55.43            | 2.60             | 2.29             | 5.71             |
| B4   | 4.02             | 91.11           | 84.63           | 43.55            | 55.44            | 1.44             | 1.06             | 6.12             |
| B5   | 6.72             | 89.39           | 82.48           | 48.06            | 57.42            | 0.00             | 10.58            | 10.58            |

Source: The authors.

Figures 4 and 5 give the evolution of the effective Young's modulus obtained by the different predictive approaches processed and the experimental results as a function of the volume fraction of the rubber fibers (reinforcement) \( V_a \) which takes the values: 0-0.1-0.2-0.25 and 0.50 for a Young's modulus is 2.214 MPa.

If we examine the results presented in Table 4 we notice that the maximum difference is reached with the combined models of Reuss and Hirsch is worth 93.66% and 88.08% respectively, this is a great value. The two models are not capable of predicting the effective behavior of the composites (soil-fiber).

As for the Popovics and Halpin-Tsai models, they gave percentages close to half, for example, with 10% fibers, this gave a difference of 42.21% and 54.29% for soil A, as for the ground B; this gave 46.83% and 54.58. %, respectively.

The Voigt, Hashin, and Bache approximations demonstrate a minimal deviation range, spanning from 0.00% to 10.66% for Voigt, 0.00% to 12.717% for Hashin, and 0.00% to 12.988% for Bache, considering two types of soils, A and B.
These three models prove effective in predicting the composite materials (soil-rubber fibers) effective Young's modulus.

Figure 4. Effective elastic moduli as a function of the volume fraction of the reinforcement comparison of analytical and experimental results for Ayaida clay.

Figure 5. Effective elastic moduli as a function of the volume fraction of the reinforcement comparison of analytical and experimental results for Bentonite de Maghnia.
5 CONCLUSION

Our comparative study allows us to conclude that:

- the rise in fiber content leads to a decrease in the dry density of both types of clay soil, primarily attributable to the lower specific gravity and unit weight of the fibers;
- the increase in rubber fiber content leads to a decrease in the modulus of elasticity in both soils A and B;
- the results obtained from predictive approaches for calculating the modulus of elasticity in two clayey soils reinforced with rubber fibers from worn tires indicate that the Voigt, Hashin, and Bache models are in closer agreement with experimental results, showing a maximum deviation of 10.66% for Voigt, 12.717% for Hashin, and 12.988% for Bache in the case of soil A. In soil B, the maximum differences are 6.72%, 0%, and 10.58% with the same models when using 50% fibers. The Voigt and Hashin models exhibit superior accuracy, particularly for clayey soils incorporating fibers.

LIMITATION OF THE STUDY AND FUTURE RESEARCH

This study involves predicting the effective mechanical properties of composites using well-validated mixing laws and is of significant practical interest in situations where testing is impossible, difficult, or expensive. In this study, some of these composite models provide accurate calculations and closer values for the moduli of elasticity. They can also estimate the modulus of elasticity for clayey soil reinforced with scrap tire rubber fibers, offering a viable alternative to previous models with the necessary assumptions. Future studies could investigate some of the composite models not examined in this paper for clay soil reinforced with rubber fibers to predict the modulus of elasticity and compare the results with experimental data. Future studies could develop composite models specifically tailored for soils, rather than relying on models primarily designed for concrete, to accurately predict the modulus of elasticity for clay composites reinforced with rubber fibers from scrap tires.
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