2D numerical modeling of crack propagation using SFEMD method of a LEHI material

Modelagem numérica 2D da propagação de trincas usando o método SFEMD de um material LEHI

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ABSTRACT
Numerical methods today play a useful and important role in solving various problems related to fracture mechanics including modeling crack propagation, fretting fatigue and cohesion... Furthermore, these methods have been widely used to solve problems in linear and nonlinear fracture mechanics in cases of elastic, plastic fracture problems. The evaluation of stress intensity factor in 2D and 3D geometries, thus these techniques widely used for non-standard crack configurations. The objective of this work is study the effects of the crack length and the number of structural elements on the two crack parameters such as the stress intensity factor $K_I$ and $K_{II}$ and the contour integral ($J$). As well as presenting a numerical modeling of crack propagation, for a LEHIM (Linear Elastic Homogeneous Isotopic Material) with mechanical characteristics by the method
SFEMD (Stretching Finite Element Method Developed), the model chosen with quadratic elements with 4 nodes (CPE4). However, this method is based on the effect of the number of contours and the number of elements around the crack tip on the variation of the stress intensity factors. In addition, the crack propagation criterion (MCSC) was used, a computer program was created in FORTRAN language to develop and evaluate the stress intensity factors and the contour integral (J). Several examples of crack lengths $a = 0.7, 1.4, 2.1, 2.8$ and $3.5$ mm were used. Additionally, the number of items has been changed several times. The stress intensity factors of modes I and II and direction angles ($\alpha$) are calculated to solve the problem using the ABAQUS finite element code. The results obtained by the SFEMD method and the results of the analytical method are very close.

**Keywords**: stress intensity factors, contour integral (J), SFEMD, crack, ASTM A36 steel material.

**RESUMO**
Atualmente, os métodos numéricos desempenham um papel útil e importante na solução de vários problemas relacionados à mecânica da fratura, incluindo a modelagem da propagação de trincas, fadiga por atrito e coesão... Além disso, esses métodos têm sido amplamente usados para resolver problemas de mecânica da fratura linear e não linear em casos de problemas de fratura elástica e plástica. A avaliação do fator de intensidade de tensão em geometrias 2D e 3D, portanto, essas técnicas são amplamente usadas para configurações de rachaduras não padronizadas. O objetivo deste trabalho é estudar os efeitos do comprimento da trinca e do número de elementos estruturais nos dois parâmetros da trinca, como o fator de intensidade de tensão $K_I$ e $K_{II}$ e a integral de contorno (J). Além de apresentar uma modelagem numérica da propagação da trinca, para um LEHIM (Linear Elastic Homogeneous Isotopic Material) com características mecânicas pelo método SFEMD (Stretching Finite Element Method Developed), o modelo escolhido com elementos quadráticos com 4 nós (CPE4). No entanto, esse método se baseia no efeito do número de contornos e do número de elementos ao redor da ponta da trinca na variação dos fatores de intensidade de tensão. Além disso, foi usado o critério de propagação de trinca (MCSC) e foi criado um programa de computador em linguagem FORTRAN para desenvolver e avaliar os fatores de intensidade de tensão e a integral de contorno (J). Foram usados vários exemplos de comprimentos de trinca $a = 0.7, 1.4, 2.1, 2.8$ e $3.5$ mm. Além disso, o número de itens foi alterado várias vezes. Os fatores de intensidade de tensão dos modos I e II e os ângulos de direção ($\alpha$) são calculados para resolver o problema usando o código de elementos finitos ABAQUS. Os resultados obtidos pelo método SFEMD e os resultados do método analítico são muito próximos.

**Palavras-chave**: fatores de intensidade de tensão, integral de contorno (J), SFEMD, trinca, material de aço ASTM A36.
1 INTRODUCTION

In fracture mechanics, the various lines of research are based on numerical simulation methods, to model or simulate crack propagation or solve other problems. In addition, linear fracture mechanics remains today the most widely used in practice. Different numerical methods exist in the field of mechanics. Among them, the stretching finite element method (SFEM) is an evolution of the classical finite element method.

This method was first developed by (Bentahar et al., 2017). Indeed, this method is assigned to each propagation of new coordinates for each node. Thus, the mesh keeps the same number of node and elements. Indeed, several studies have been proposed in the literature to optimize and evaluate the two characterization parameters of the stress field near the bottom of the crack. Especially the last years among them, (Breitbarth et al.,) summarized a procedure to calculate J-integral and stress intensity factor $K_I$ and $K_II$ based on (DIC) data. (Gajjar and Pathak, 2021) used the extended finite element method to study the influence of the graded property of plasticity and thermal boundary conditions on the contour integral ($J$). On the other hand, (Lepretre et al., 2021) used the finite element method to deal with the) SIF) evaluation of a repaired cracked steel plate using semi-empirical analysis. General solutions of stress intensity factors (SIF) have been presented for the problem of an edge-like crack under uniform mechanical and thermal loads over a given distance (Chen et al., 2021). There are other methods that helped evaluate the stress intensity factor (SIF) and J integral for centred crack on an elastoplastic material (Bentahar, 2023) and the initial crack propagation problem (Bentahar; Benzaama, 2023; Bentahar, 2024).

The SIF was calculated from the J-integral, using the finite element method by (Toribio et al., 2021; Christer; Kjell, 2021) extended a work to include the area integral ($J$) by deriving it as a function of displacements only, to also obtain the alternative method of calculating ($J$) in peri-dynamics. Therefore, the stress intensity factor at crack tip were calculated by the stress extrapolation method (Luo et al., 2021) such as the stress intensity factor ($K_i$, $K_{ii}$) and the integral of the contour ($J$). Among these studies we cite the work (Shuai et al., 2021) created a modified J-integral calculation method is adopted to solve the problem of evaluating crack propagation of shot-blasted structures. (Sutthisak et al., 2008)
presented an adaptive finite element method for two-dimensional and axisymmetric nonlinear analysis that uses the contour integral \((J)\) as a parameter, to characterize the severity of stresses and strains near the crack tip. \((\text{Achchhe et al.}, 2021)\) studied by the extended finite element method, the crack growth and the energy release rate \((\text{ERR})\), of a cracked plate at the edge under different loads like tension and stress. \((\text{Gozin and Aghaie-Khafri, 2012})\) presented a study based on the concept of plasticity-induced crack closure \((\text{PICC})\) and used the method of elements \((\text{FEM})\) to study the effect of the compressive residual stress field on the crack growth from a hole.

Another Petrov-Galerkin natural element \((\text{PG-NE})\) method based on mixed modes has been proposed by \((\text{Cho, 2015})\) to evaluate the stress intensity factors of a tilted \((2D)\) crack.

A comparative study between the finite element method \((\text{FEM})\) and the analytical method was presented by \((\text{Murat, 2016})\) to solve the problems of a stratified composite plane containing an internal perpendicular crack. \((\text{De Morais, 2007})\) estimated the set of SIF values and applied the elasticity relations to obtain the displacement jumps along the crack faces. A new bill criterion based on the concept of maximum potential energy release rate \((\text{MPERR})\) was applied by \((\text{Chang et al., 2006})\) to predict the direction of failure. \((\text{Anlas et al., 2000})\) used several different numerical techniques like the element-free Galerkin method \((\text{EFGM})\) of a fissured plate to compare and calculate the SIF. To model the fracture, the split knot strategy is used, and an increase in the crack follows the path of successive linear extensions \((\text{Alshoaibi and Fageehi, 2023})\). \((\text{XFEM})\) is often used due to its advantages, like the capability of allowing the crack propagation through the elements without the need of remeshing after each time step, reducing the computation cost \((\text{Nikfam et al., 2019})\). The reduction of the opening of the crack tip is an indication that the stress advances effectively the crack is low \((\text{Alderliesten, 2005; Anindito; Makabe, 2009})\).

The two most used methods to calculate the \((\text{SIF})\) in fracture mechanics the displacement extrapolation technique \((\text{Zhu; Smith, 1995; Guinea et al., 2000})\) and the Integral \((J)\) method \((\text{Rice, 1968; Courtin et al., 2005; Fageehi; Alshoaibi 2020})\).
In fracture mechanics it is easy to obtain the deformation field already present near the crack end and on the other hand can directly reflect for example the effects of crack closure or plasticity (Breitbarth et al., 2019).

The fatigue life of the stable stage could be calculated by the crack propagation law (Niu et al., 2023). The stress intensity factor (SIF), is the only significant parameter that controls and characterizes the stress field near, the crack tip of a fracture propagation problem (Farrokh; Jon, 2012).

To evaluate the stress intensity factors (SIF), it is possible to use an isometric analysis in order to control the crack propagation (Montassir et al., 2023). Another type of work can also be found in crack analysis using SIF (Ismail et al., 2011; Ismail et al., 2014).

The aim of this study is to model by the analysis the variation of the crack parameters. Specifically, the evaluation of the stress intensity factor, contour integral ($J$) and the ratio ($K_{I}/K_{0}$) for a model in mode (I) by the SFEMD method of an ASTM A36 steel material. The comparison was also made in other methods. In addition to the effect of changing the total number of the mesh elements and the effect of crack length on crack properties by Abaqus software.

2 ASPECT OF FRACTURE MECHANICS
2.1 STRESS FIELD AROUND THE CRACK TIP

\[
\sigma_{ij}^{I,II}(r, \theta) = \frac{K_{I,II}}{\sqrt{2\pi r}} f_{ij}(\theta)
\]
Where

\[ K_{I, II} \] is the SIF in mode I and II,
\[ \sigma_{ij}^{II} \] is the stress field associated with mode (I).

Tada \textit{et al.} (2000) gave the general equation of the stress field in 2D in the vicinity of the crack front defined by the stress intensity factor K. Thus (Sih; Erdogan, 1962).

Have define that the crack propagate according to the direction perpendicular to the maximum tangential.

\[ \sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} (1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}) \]

\[ \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} (1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}) \]

\[ \tau_{xx} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \]

\[ 2.2 \text{ CHARACTERIZATION PARAMETERS (STRESS INTENSITY FACTORS)} \]

In fracture mechanics, the characterization parameters play a beneficial role, in particular the stress intensity factor K. (Saverio, 2014) defined SIF as the only significant parameter, which makes it possible to know the stress and strain state at any fault point. On the other hand, the analytical stress intensity factor was given by (Ewalds; Wanhill, 1989).

\[ K_I = F \sigma \sqrt{a\pi} \] (3)

Where

F is the correction factor given by the following relationship:

\[ F = 1.12 - 0.23 \left( \frac{a}{w} \right) + 10.6 \left( \frac{a}{w} \right)^2 - 21.7 \left( \frac{a}{w} \right)^3 + 30.4 \left( \frac{a}{w} \right)^4 \] (4)
Where

The stress intensity factor $K_{II}$ calculated by the relation:

$$K_{I} \sin \theta + K_{II}(3 \cos \theta - 1) = 0 \quad (5)$$

The contour integral $(J)$ in the case of the plane stresses of a homogeneous and isotropic material is given by the relation:

$$J = \frac{1}{E}(K_{I}^2 + K_{II}^2) \quad (6)$$

Where:

$E$ : Young's modulus.

In order to independently obtain the stress intensity coefficients in the different modes, decoupling methods are applied which make it possible to break down the energy quantities linked to modes I and II, from which:

$$J = J_{I} + J_{II} \quad (7)$$

With:

$$J_{I} = \frac{K_{I}^2}{E}, J_{II} = \frac{K_{II}^2}{E} \quad (8)$$

Several authors developed the contour integral $(J)$ (Sih, 1968; Nguyen, 1980; Destuynder and Djaoua, 1981) by introducing an arbitrary field in the formulation of the integral they approached.
3 PROGRAM INTERFACE (FORTRAN/ABAQUS)

The execution of the FORTRAN program with the ABAQUS software interface of the finite element method, to calculate \( K_I \), \( K_{II} \) and contour integral \( J \) concerning the various modeling problems. Regarding the effect of changing the total number of elements and the effect of crack length as shown in Figure 3.

Figure 3. Flowchart of the optimization of \((K_I, K_{II} \text{ and } J)\) of a crack problem by the SFEMD method

Source: Authors.
4 MODELING MODEL

The geometry of the cracked mesh, in our work is illustrated in Figure 4 the mesh considered has a length \( B = 10 \text{ mm} \) and a width \( W=8\text{mm} \), the horizontal crack length is \( a = 3.5 \text{ mm} \), the mesh in homogeneous material with linear and isotropic elastic behavior. The steel structure ASTM A36, with a modulus of elasticity \( E=200\text{GPa} \) and Poisson’s ratio \( \nu=0.26 \), is subjected to a uniform tensile stress \( \sigma=120 \text{ MPa} \). The ABAQUS finite element code was applied to optimize the stress state at the crack tip (SIF) and the contour integral (J). The FORTRAN program was used to create a structured mesh, of the CPE4 type with four nodes, consisting of 478, 524, 570, 616 and 662 square elements and 508 nodes.

![Figure 4. Mesh geometry. (a) initial crack and boundary conditions; (b) SFEM method](image)

5 RESULTS AND VALIDATIONS

![Figure 5. Discretization of the initial crack: (a) meshless method (Rao and Rahman 2000) and (b) SFEMD method](image)
Figure 6. Discretization of crack propagation with trajectory: (a) the meshless method (Rao and Rahman 2000) and (b) SFEMD method

Figure 7. Evolution of the stress intensity factor as a function of number of contours between the SFEMD method and the results (Rao and Rahman 2000) and the analytical method of ASTM A36 steel material

Figure 7 shows the evolution of the stress intensity factors ($K_i$ and $K_{II}$) as a function of number of contours, regarding the ASTM A36 steel material, we can notice that the change of the $K_i$ between the values 30 and 40MPa(m)$^{1/2}$, on the other hand, the values of the ($K_{II}$) change between 4 and 5 MPa(m)$^{1/2}$, as well as the values of ($K_i$) are greater compared to the other values of ($K_{II}$), in addition, the results obtained are proportional between the different validation methods.

5.1 ESTIMATION OF THE RATIO ($K_i/K_0$)

The comparison of the ratio ($K_i/K_0$) between the SFEMD method and other methods such as the study proposed by (Aliabadi, 2002; Erdogan and Civelek, 1982) compared to the analytical method. This estimate made in the case of the ratio $a/w = (0.2, 0.3, 0.4$ and $0.5)$, as shown in Table1.
Table 1. Different values of the ratio \((K_I/K_0)\) regarding the different methods: (Aliabadi, 2002; Erdogan and Civelek, 1982), analytical method and SFEMD

<table>
<thead>
<tr>
<th>(a/w)</th>
<th>SFE MD Error</th>
<th>Aliabadi 2002 Error%</th>
<th>Erdogan 1982 Error</th>
<th>Analytical solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.60 -16.6</td>
<td>1.57 -14.6</td>
<td>1.49 -8.7</td>
<td>1.37</td>
</tr>
<tr>
<td>0.3</td>
<td>2.00 -17.6</td>
<td>1.96 -15.3</td>
<td>1.85 -8.8</td>
<td>1.70</td>
</tr>
<tr>
<td>0.4</td>
<td>2.48 -16.9</td>
<td>2.32 -5.2</td>
<td>2.32 -9.4</td>
<td>2.12</td>
</tr>
<tr>
<td>0.5</td>
<td>2.93 -3.16</td>
<td>3.27 -15</td>
<td>3.01 -6</td>
<td>2.84</td>
</tr>
</tbody>
</table>

Source: Authors.

Figure 8. Estimation of the ratio \((K_I/K_0)\) between the SFEMD method and the results obtained (Aliabadi, 2002; Erdogan and Civelek, 1982) and the analytical method with \(a/w\) =0.2, 0.3, 0.4 and 0.5 of ASTM A36 steel material.

![Figure 8](image)

Source: Authors.

The results obtained are compared with those published (Aliabadi, 2002; Erdogan; Civelek, 1982) and the analytical method. Four cases of the ratio \(a/w\) were taken into account \(a/w = (0.2, 0.3, 0.4, \) and \(0.5)\). \(K_0\) is the exact solution used to normalize the stress intensity factors before the geometric correction, and \(K_I\) the stress intensity factor after the geometric correction. In the case of the ratio \(a/w =0.2\) the comparison represents an error of 16.7%, 14.6% and 8.7% between the analytical method, SFEMD method, (Aliabadi, 2002; Erdogan; Civelek, 1982) respectively. In the case of the ratio \(a/w =0.3\) the comparison represents an error of 17.6%, 15.3% and 8.8% the third comparison for \(a/w =0.4\), which presents an error of 16.9%, 5.2% and 9.4%. For the last validation of the ratio \((K_I/K_0)\), in the case of the ratio \(a/w =0.5\) compared to the reference values of the analytical method, the following results are obtained: 3.16%, 15% and 6% respectively. The SFEMD method gives results similar to the results obtained by the methods proposed by (Aliabadi, 2002; Erdogan; Civelek, 1982) and compared by the analytical method. After the figure 7 we can notice that the increase in the ratio
(a/w) causes an increase in the value of the ratio (Ki/K0), and the results found for the estimation of the error are proportional to the analytical method.

5.2 THE EFFECT OF VARYING THE TOTAL NUMBER OF ELEMENTS

We propose to model the mesh with various numbers of elements, 478, 524, 570, 616, and 662, this comparison is carried out with the analytical method as shown in Figure 9.

![Figure 9. SFEMD method for different number of elements](source)

Figure 10 shows the comparison of the stress intensity factor Ki and Kii, between the method (SFEMD) and the analytical method for different elements (478, 524, 570, 616 and 662). This is in agreement with the results established by the analytical method. The results obtained allow us to conclude that there is a
very good agreement between the results obtained by the two analytical and numerical methods.

5.3 THE EFFECT OF CRACK LENGTH

We consider a structure with different initial crack lengths, in the cases of $a=0.7\text{mm}$, 1.4, 2.1, 2.8 and 3.5mm as shown in Figure 11.

Figure 11. SFEMD method for different crack lengths: (a) $a=0.7\text{mm}$, (b) $a=1.4\text{mm}$, (c) $a=2.1\text{mm}$, (d) $a=2.8\text{mm}$ and (e) $a=3.5\text{mm}$ of ASTM A36 steel material.

![SFEMD method](image)

Source: Authors.

Figure 12. Evolution of the stress intensity factor as a function of the ratio ($a/w$) of an initial crack for different crack lengths $a=0.7$, 1.4, 2.1, 2.8 and 3.5: a) $p=0$, b) $p=1$, c) $p=2$, d) $p=3$, e) $p=4$ and f) $p=5$ of ASTM A36 steel material.

![Evolution of stress intensity factor](image)

(a) ![Graph](image)

(b) ![Graph](image)
Figure 12 shows the evolution of the SIF according to the different ratios a/w= 0.1, 0.2, 0.3, 0.4 and 0.5. We note that the increase in the ratio (a/w) causes an increase in the SIF in all cases of crack propagation, example for p=1 (a/w=0.5, K_I= 30.73 and K_II=4.128, a/w=0.4, K_I = 25.08 and K_II =3.452, a/w=0.3, K_I = 17.37 and K_II =2.117, a/w=0.2, K_I = 12.14 and K_II= 0.9689, a/w=0.1, K_I=11.56 and K_II=0.8508). On the other hand, an inverse relationship between K_I and K_II is shown, which means that the more the stress intensity factor K_I increases the more the stress intensity factor K_II decreases with the increase of the ratio (a/w). This is indicated by numerous researches among them the analytical solution proposed by (Ewalds and Wanhill, 1989)in the case of the variation of the stress intensity factor (K_I) as a function of (a/w) (Nguyen et al., 2023; Fayed, 2017) and the stress intensity factor of (K_I) as a function of length (a) (Wang et al., 2018; Fageehi, 2022).
Figure 13. Evolution of the J-integral as a function of the ratio (a/w) of an initial crack for different crack lengths a = 0.7, 1.4, 2.1, 2.8 and 3.5 of ASTM A36 steel material.

Figure 13 presents the evolution of (J) according to the different ratios a/w = 0.1, 0.2, 0.3, 0.4 and 0.5. It can be seen that the values of contour integral (J) are proportional with the values of $K_{I}$. $p=0$ (Figure 13a), $p=1$ (Figure 13b), $p=2$ (Figure 13c), $p=3$ (Figure 13d), $p=4$ (Figure 13e) and $p=5$ (Figure 13f). It can be seen that the J-integral values vary between 0 and $4.0 \times 10^{-3} \text{KJ/m}^2$. Moreover, the results obtained are acceptable compared to the results of other researches obtained.
concerning the contour integration (J), and among these works we obtain the research works proposed by (Bentahar et al., 2021; Bentahar; Benzaama, 2023).

Concerning the propagation of crack in fretting fatigue (Gajjar; Pathak, 2021) in the case of the fracture analysis of a graded plastic material with thermomechanical integration (J), indeed, all the research in this field confirms that the greater the ratio ($a/w$) is increased, the more the integral of the contour (J) is increased.

6 CONCLUSION

Figure 14. Evolution of the stress intensity factor $K_I$ and $K_{II}$ as a function of the ratio ($a/w$) for different ratios for all cases of crack propagation $p= 1, 2, 3, 4$ and $5$ of ASTM A36 steel material

Numerical analysis by the stretching finite element method (SFEMD) has been used successfully for linear-elastic fracture mechanics problems of the (ASTM A36) steel material.

The proposed method gives results similar to those obtained by the analytical solution, this has been verified on a case where the analytical solution is established.

The stress intensity factor is evaluated by several analyses, the finite element method and its results are in good agreement with those of the exact solution. The results of the present approach provide advantageously comparable values and show their effectiveness in modeling crack propagation problems. In the case where ($a/w$) = 0.1 to 0.5 with the analytical method, the criterion of (MCS) used to calculate the angle of orientation gives good results and a good correlation
was obtained between the SFEMD method, the method (Rao and Rahman 2000) and the analytical method. On the other hand, the estimate of the error on the stress intensity factors for mode (I) is less than 4.591%, it is evaluated as an acceptable threshold. We obtained a good correlation between all the simulation results between the two SFEM methods and the analytical method.

In academic circles, this method is one of the existing numerical methods that helps students and researchers to simulate using software. It is a complementary method to the finite element method, which is famous for researchers in the field of technology in general and in the field of mechanics in particular. The results obtained help researchers how to maintain the number of elements present in the structure without changing their number, which may affect the mechanical structure of the metal during the presence of crack propagation.

Moreover, the proposed method can be the beginning of an effective research approach for scholars and researchers in the field of fracture mechanics, especially researchers in the field of fatigue, fracture and fretting fatigue in composite and non-composite materials which have different mechanical properties.

Finally, we can say that numerical methods have an essential place in the modeling and simulation of cracks in structures in order to avoid cracking problems at the plate level. Furthermore, the method presented here in our work plays a fundamental role in the simulation. This is a new method based mainly on the SFEMD method, itself based on the well-known FEM method. The method presented is also based on modeling, the effect of crack propagation length and number of structural elements on crack parameters such as stress intensity factor $K_I$ and $K_{II}$ and contour integral $J$. 
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