Use of optimization methods in the calculation of slope stability

Uso de métodos de otimização no cálculo da estabilidade de taludes

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ABSTRACT
This article focuses on studying and identifying factors that contribute to ground movement and determining appropriate methods for verifying slope stability. Depending on the type of soil, the geometry of the slope and the type of work to be carried out, it is appropriate to choose a suitable technical solution, adapted both to the nature of the soil in place and to its environment. The major techniques that can be used to improve the mechanical properties of soils are the modification of the internal structure of the existing soil and the reinforcement of the soil by the addition of materials and inclusions. These techniques make it possible to improve the compactness and bearing capacity of the soil in place. The case presented in this research was studied using a two-dimensional finite element model, to study the effect of various mechanical characteristics of the soil, including cohesion (C), friction angle (φ), the reference module (Eref) and the backfill volume. We examined the influence of the coupling on the safety factor and the location of the sliding surface to have an optimal slope design. To do this, we formulated the problem into an optimization framework using MINITAB 19 software. This framework will use a design of experiments (DOE) approach to identify the optimal
combination of factors ensuring slope safety. The results obtained will be analyzed to evaluate the effectiveness of the DOE method in determining the most favorable combination of parameters. We expect the mechanical properties of the slope to have the most significant influence on the factor of safety and the location of the sliding surface.

**Keywords:** safety factor, Plaxis 2D, optimization, Taguchi method, ANOVA, response surface method.

**RESUMO**
Este artigo se concentra no estudo e na identificação dos fatores que contribuem para o movimento do solo e na determinação dos métodos adequados para verificar a estabilidade do talude. Dependendo do tipo de solo, da geometria do talude e do tipo de trabalho a ser realizado, é apropriado escolher uma solução técnica adequada, adaptada à natureza do solo no local e ao seu ambiente. As principais técnicas que podem ser usadas para aumentar as propriedades mecânicas dos solos são a modificação da estrutura interna do solo existente e o reforço do solo com a adição de materiais e inclusões. Essas técnicas possibilitam melhorar a compactação e a capacidade de suporte do solo no local. O caso apresentado nesta pesquisa foi estudado por meio de um modelo bidimensional de elementos finitos, para estudar o efeito de várias características mecânicas do solo, incluindo a coesão (C), o ângulo de atrito (φ), o módulo de referência (Eref) e o volume de aterro. Examinamos a influência do acoplamento sobre o fator de segurança e a localização da superfície de deslizamento para obter um projeto de talude ideal. Para isso, formulamos o problema em uma estrutura de otimização usando o software MINITAB 19. Essa estrutura usará uma abordagem de projeto de experimentos (DOE) para identificar a combinação ideal de fatores que garantam a segurança do talude. Os resultados obtidos serão analisados para avaliar a eficácia do método DOE na determinação da combinação mais favorável de parâmetros. Esperamos que as propriedades mecânicas do talude tenham a influência mais significativa sobre o fator de segurança e a localização da superfície de deslizamento.

**Palavras-chave:** fator de segurança, Plaxis 2D, otimização, método Taguchi, ANOVA, método de superfície de resposta.

**1 INTRODUCTION**
Over the past few decades, many studies have attempted to calculate the slope safety factor, an important indicator for determining whether a slope is stable and therefore can help prevent landslides. Several methods have been proposed (Fellenius *et al.*, 2023; Wang *et al.*, 2024; Janbu *et al.*, 1973; Zhang *et al.*, 2024; Milena *et al.*, 2021). These methods calculate the slope safety factor using the limit equilibrium method (LEM) (Fredlund *et al.*, 1977). However, these methods only solve the minimum safety factor problem for a known sliding surface. On the other
hand, numerical calculation methods, such as the finite element method and the finite difference element method, can simulate physical behaviors using computer tools without the need to simplify the problem, are considered affordable, potent and viable alternatives, (Nansheng et al., 2015). Thus the reduction in shear strength (SSR) (Dawson et al., 1999; Griffiths et al., 1999; Matsui et al., 1992; Zienkiewicz et al., 1975; Smith et al., 2013; Sun et al., 2017). The full slope analysis requires that the critical failure surface (CFS) corresponding to the minimum factor of safety is one of the probable test failure surfaces. Many researchers have introduced different minimization procedures to calculate the minimum safety factor. These classic minimization/optimization procedures are: the variation methods of (Baker et al., 1978; Baker et al., 1980), Simplex method of (Chen et al., 1988; Nguyen, 1985), Alternating variable method by (Celestino et al., 1981), Conjugate gradient method by (Arai et al., 1985), these conventional methods are simple and fast. However, classical optimization methods are susceptible to being trapped by the desired result converging to a local minimum due to considering a lower number of trial failure surfaces. On the other hand, if too many trial failure surfaces are considered, it makes the search computationally inefficient in terms of computer memory allocation and execution time.

Considering the limitations of the conventional methods mentioned above to predict the factor of safety, real global minimization methods have been adopted in the last decades. Several approaches inspired by heuristic algorithms have been developed to integrate the learning capacity in the search for the procedure of this unique surface with the determination of the optimal combination parameters and the minimization of the objective function, the goal is to achieve an optimal design that is safe, robust and cost effective.

In the field of geotechnical engineering, researchers have applied advanced optimization techniques for different purposes: finding the best design with respect to geometry, shape, weight and cost...etc. The geometric parameters adopted for the slopes such as their height, inclination, length and mechanical properties for stability present the main considerations allowing engineers to place the problem in an optimization framework and decide whether an optimal design will be appropriate. in terms of stability and economy.
Recently, optimization techniques such as artificial neural networks (ANN), genetic algorithms (GA), particle swarm optimization (PSO), ant colony optimization (ACO), and design of experiments (DOE) such as Taguchi method, Response Surface Methodology (RSM) and Factorial Design have been widely applied in many geotechnical optimization problems, each optimization technique has its advantages and disadvantages. Among these research works we cite: (Simpson et al., 1993; Benayoun et al., 2020; Goh, 2000; McCombie et al., 2002; Kar et al., 2022). Slope stability reinforced with nails has also been the subject of several studies (Mangnejo et al., 2019; Mazouz et al., 2022; Benayoun et al., 2021), studied the optimization of soil behavior parameters based on the Design of Experiments (DOE) method.

In this study, the problem of optimizing the geometric and mechanical parameters of the slope is addressed, the variability of soil properties having a significant influence on the stability of the structure. The goal is to find the best combination that meets our requirements and results in an optimal design. The first step is the identification of factors favoring ground movements using the Plaxis calculation framework, the effect of mechanical characteristics of the ground such as cohesion (C), friction angle (φ) and Eref module is taken into account. Then, a presentation of the optimization approach for these different parameters with their influence on the safety factor was made.

2 MODELING OF THE EMBANKMENT BY FINITE ELEMENTS

2.1 THE GEOMETRY OF THE NUMERICAL MODEL

Figure 1 shows a multi-layered embankment. For the stability calculation, we modeled this embankment numerically, using the two-dimensional finite element software Plaxis2D version 8.2, with a plane strain problem and a long-term behavior using analysis conditions drained and the Mohr-Coulomb model. The model requires five basic input parameters: Young's modulus E, Poisson's ratio ν, cohesion C, angle of friction φ and angle of dilatancy ψ. In the geometric model, the mesh was made by triangular elements of 15 nodes at high precision, to generate a finite element mesh. The left and right limits of the model were fixed in the horizontal direction while the lower limit was fixed in all directions.
Several studies have been performed on the effect of parametric variation on slope stability and excavation, such as loads, excavation height, inclusion dimensions, mechanical characteristics...etc.

In our case, the effect of the variation of the mechanical and geometric properties on the factor of safety was studied. The aim of the choice of optimization techniques is to find the optimal combination of slope input factors.

The safety factor was calculated using the phi-reduction technique available in Plaxis2D, in which the shear strength parameters are reduced in steps until the soil body fails as shown in Equation 1.

$$\sum MSF = \frac{\tan \varphi_{\text{input}}}{\tan \varphi_{\text{reduced}}} = \frac{C_{\text{input}}}{C_{\text{reduced}}}$$  \hspace{1cm} (1)

Where:
∅_{\text{input}}: the input value of the angle of internal friction (°)
∅_{\text{reduced}}: the reduced value of the angle of internal friction at failure (°)
C_{\text{input}}: the input value of the cohesion (kPa)
C_{\text{reduced}}: the reduced value of cohesion at break (kPa).

MSF: Safety factor for simulations.

Table 2. Selected parameters and levels

<table>
<thead>
<tr>
<th>№</th>
<th>Parameter</th>
<th>Unit</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>kN/m²</td>
<td>25</td>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>(\phi)</td>
<td>Degree</td>
<td>30</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>(E_{\text{ref}})</td>
<td>kN/m²</td>
<td>500</td>
<td>10000</td>
<td>15000</td>
</tr>
</tbody>
</table>

Source: Authors

2.2 MATHEMATICAL MODEL

The simulation of input-output data collected after modeling is used to establish the relationship between the input factors and the response variable according to a modeling algorithm by combining techniques such as regression model. It is necessary to determine the appropriate function, and the emphasis is now on the nature of the relationship between the response and the factors, rather than identifying the important factors.

The regression equation is calculated by the average values of the safety factor under different input parameter conditions.

3 OPTIMIZATION TECHNIQUES

3.1 TAGUCHI DESIGN OF EXPERIMENTS

Design of experiments (DOE) is one of the best-known optimization techniques. In the 1920s in England, Ronald Aylmer Fisher introduced a powerful statistical technique to study the effect of multiple variables simultaneously. In the late 1940s, Dr. Taguchi standardized a version of DOE, popularly known as the Taguchi Method. In the early 1980s it was introduced to the United States. Here are five types of design of experiments: (1) screening design, (2) factorial design, (3) response surface method (RSM) design, (4) mixture, (5) Taguchi.

DOE using Taguchi’s approach has become a much more attractive tool for practicing engineers and scientists. It is a systematic method for determining the relationship between factors affecting a process and the outcome. In other words, it is used to find cause and effect relationships. This information is needed to manage processing inputs to optimize output. The DOE can show how to perform the fewest experiments while retaining the most important information.
Taguchi’s experimental designs, often referred to as orthogonal (OA) matrices, use signal-to-noise (S/N) ratio as a measurable value of choice quality characteristics (Cheng et al., 2007). When using the Taguchi method with three levels, an L27 or L18 orthogonal lattice is most commonly used.

Table 3. L27 orthogonal design array and measured responses and s/n ratios

<table>
<thead>
<tr>
<th>Nº</th>
<th>Input Factor</th>
<th>Response FS</th>
<th>Ratio S/N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>( \phi )</td>
<td>Ref</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
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<td>500</td>
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<td>27</td>
<td>50</td>
<td>40</td>
<td>15000</td>
</tr>
</tbody>
</table>

Source: Authors

Using Minitab19, each output parameter is statistically analyzed and the main effects plots of the signal-to-noise (S/N) ratios for the output measurements are obtained.
Figure 2. Main effects plot for S/N ratios

![Main Effects Plot for S/N ratios](image)

Source: Authors

Figure 2, shows the influence of various input parameters on stability from main effects plot for S/N ratios, if the line of a parameter is almost horizontal, the parameter has no significant effect. On the other hand, a parameter for which the line has the strongest inclination has the most significant effect. In this case, the cohesion has the greatest influence on the stability, followed by the angle of friction, while the modulus $E_{ref}$ has the least influence.

<table>
<thead>
<tr>
<th>Level</th>
<th>C</th>
<th>$\varphi$</th>
<th>$E_{ref}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.62</td>
<td>-1.896</td>
<td>-2.271</td>
</tr>
<tr>
<td>2</td>
<td>-2.48</td>
<td>-2.346</td>
<td>-2.26</td>
</tr>
<tr>
<td>3</td>
<td>-2.697</td>
<td>-2.555</td>
<td>-2.266</td>
</tr>
<tr>
<td>Delta</td>
<td>1.077</td>
<td>0.658</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Source: Authors

Table 4. Answers for "smaller is better" signal to noise ratio

It clearly emerges from Figure 2, that the minimum safety factor is reached at the combination of the control parameters 5 kN/m², 30° and 10000 kN/m² which respectively represent the cohesion, the angle of internal friction and $E_{ref}$. The optimal combination (A1-B1-C2) was selected with the highest signal-to-noise ratio.

In Table 4, the ranking of the process parameters is obtained by Minitab using the signal-to-noise ratios and for different parameter levels for the safety factor. The last boxes at the bottom indicate the rank of the entries according to the results. The ranks indicate that the most impacting input parameter is cohesion (C), the second in terms of impact is the angle of friction ($\varphi$) and the third is $E_{ref}$, which strongly agrees with the above results.
The interaction graph between the process parameters such as the cohesion, the angle of friction and the modulus Eref are also shown in Figure 3. We can see that there is a substantial interaction between the cohesion and the angle of friction, while a moderate interaction between cohesion, Eref and between the angle of friction also.

![Figure 3. Interaction diagram for S/N ratios](image)

Source: Authors

3.2 SAFETY FACTOR ANALYSIS OF VARIANCE (ANOVA)

ANOVA is a statistical technique used to find the most significant process parameters that will affect the output parameters. The S/N values of the response variables are used to obtain an ANOVA using Minitab 19 software. The ANOVA examination is performed with a level of certainty (confidence) of 95% and a level of significance of 5%.

Using ANOVA analysis, it is possible to assess the significance of the selected regression model. The main idea is to compare if the residuals present a normal distribution.

A multiple regression model is adopted as shown in equation (2), it is used as the objective function (regression equation):

\[
F_s = 1.3015 - 0.0916 C - 5 + 0.0290 C - 15 + 0.0626 C - 50 - 0.0503 \varphi - 30 + 0.0096 \varphi - 35 + 0.0406 \varphi - 40 + 0.0007 \text{Eref} - 500 + 0.0008 \text{Eref} - 10000 + 0.0001 \text{Eref} - 15000
\]

(2)
From the regression model, the obtained R-squared value is 70.80%. This value is high enough to show good agreement and great importance of the predicted model. The standard deviation S is used to evaluate the efficiency of the regression model, S is equal to 0.0568 which represents the distance between the data values and the fitted values, it clearly confirms that the model can definitely predict the factor well of security.

ANOVA calculates quantities such as degrees of freedom (DF), sums of squares (Seq SS), ratio (F), value (P), as shown in Table 5.

If the P value is less than 0.05, the process parameter is said to be significant and if the P value is greater than 0.05, the process parameters are considered insignificant.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion (C)</td>
<td>2</td>
<td>0.118355</td>
<td>18.31</td>
<td>0.00</td>
</tr>
<tr>
<td>Friction angle (φ)</td>
<td>2</td>
<td>0.038425</td>
<td>5.94</td>
<td>0.009</td>
</tr>
<tr>
<td>Eref</td>
<td>20</td>
<td>0.064656</td>
<td></td>
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<tr>
<td>Total</td>
<td>26</td>
<td>0.221447</td>
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Source: Authors

The ANOVA results presented in Table 5 reveal that the most effective parameter is cohesion (C) as shown by its low P-value (P<0.05) and high F-value, followed by friction angle (φ) with a considerably low p-value and F-value, and finally Eref.

Residual plots are used to examine the goodness of fit of the model. Minitab provides the following residual plots.

The interpretation of each residual graph is given below:
The normal probability plot as shown in Figure 4 reveals that almost all the residuals follow a linear pattern and there do not seem to be any outliers, this agrees well with the results.

The residuals versus the fitted values (Figure 5) indicate that the points appear randomly scattered across the plot, there is a non-linear relationship and no outliers are apparent, the residual is mostly cumulative around zero, except for a few dots above.
Table 6. Experimental results according to a Box Behnken Design

<table>
<thead>
<tr>
<th>Case</th>
<th>Cohesion</th>
<th>Friction Angle</th>
<th>Eref</th>
<th>Experimental response variable</th>
<th>Estimated response variable</th>
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<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>30</td>
<td>7750</td>
<td>1.063</td>
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<td>35</td>
<td>7750</td>
<td>1.345</td>
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</tr>
</tbody>
</table>

Source: Authors

Figure 6. Residuals with respect to data order

Figure 6 represents the diagram of the residues according to the order. The residuals are randomly arranged around the center line and then are evenly arranged. The plot clearly shows that residuals close to each other are correlated and therefore not independent. This means that the model is valid.

3.3 APPLICATION OF THE RESPONSE SURFACE METHOD (RSM)

Response Surface Methodology (RSM) was introduced by (Box et al., 1951), in the early 1950s, it is a set of statistical and mathematical techniques useful for process development and optimization. The most used RSM technique is the Box-Behnken (BBD) design (Benayoun et al., 2021). The process was carried out according to the Box-Behnken design tool of RSM using Minitab 19
software for 3 selected factors (Table 1), in order to solve this problem of safety factor minimization. Based on the simulations performed and the results obtained using finite element analysis with Plaxis 2D, 15 tests were carried out according to the Box-Behnken design (Table 6).

In this study, the Box-Behnken (BBD) design was performed with a total of 15 experiments. Based on the Taguchi method, we could have used 27 experiments (L27), but thanks to the Box-Behnken design, the number was reduced to 15 experiments with very satisfactory results.

The results are analyzed so that the conditions can be optimized to give the best result for all response parameters. The results of the calculation of the safety factors are summarized in Table 5.

The adequacy of the quadratic response surface model was justified by ANOVA. The two parameters: cohesion and friction angle have a positive standardized effect on the safety factor (Figure 7).

![Figure 7. Main effects](source: Authors)

The Pareto chart was developed to compare the relative magnitude of the effects of various factors on the response, their significance and their interaction. Minitab plots the effects in descending order of the absolute value of the standardized effects and draws a reference line on the graph.

As shown in Figure 8, the Pareto bar of input effect A (cohesion) and AB (cohesion interaction and angle of friction), are to the right of the vertical red line; therefore, this bar is statistically significant at the 5% significance level with the
current model terms. Although AA, Bare average significant, while the other factors C, CC, BB, BC, AC seem insignificant.

The lowest total numerical experience was used to model the quadratic polynomial equation. The regression equation developed, as shown below, represents the quantitative effect of the input factors and their interactions. Therefore, a good agreement between the experimental and predicted values confirms the validity of the model (Table 6).

\[
Fs = -0.17 + 0.02835 C + 0.0551 \varphi - 0.000003 E_{ref} - 0.000108 c^2 - 0.000465 \varphi^2 - 0.000549 c \times \varphi
\]  

(3)

To determine the magnitude, direction, and significance of effects, we can use the normal probability plot of effects. In the graph of Figure 9, the main effect of the factor A (cohesion) and AB (cohesion and Angle of friction) on the factor of safety is statistically significant at the level of 0.05. This point has a distinct color and shape (red square) from the points for insignificant effects (blue circle). The normal plot of the standardized effects, proved that the most important factors on the factor of safety are the cohesion and the cohesion-Angle of friction.
3.4 OPTIMIZATION USING RESPONSE LEVEL CURVES

The relationship between factors and responses is well understood when using contour lines. Contour plotting provides a 2D view of the surface where points that have the same response are connected to produce contour lines of constant responses. In Figures 10 a-b-c, the level curves show the response as a function of the other variables: cohesion, Angle of friction and Eref. The plots are very explicit as to the influence of the parameters on the safety factor. We clearly see the increase in the factor of safety with the increase in the cohesion and Angle of friction. On the other hand, Eref has little effect on the safety factor (Figure 10c).
The effects of the parameters on the safety factor were analyzed by plotting three-dimensional (3D) response surface graphs. The response is presented in a continuous curved surface, in 3D space depending on the parameters of interest. The variation of the two parameters are included in the test, while the third parameter remains unchanged.

The plots visualize the response and the interactions between the two chosen parameters. For example, the response surfaces are presented in Figure 11. Note the following unchanged parameters: 7750 kPa of Eref (Figure 11a), 27.5 kPa of cohesion (Figure 11b) and 35° of angle of friction (Figure 11c) respectively. The plot of the combined cohesion-friction angle effect at 7750 kPa of Eref is shown in Figure 11a, shows the interactive effects of cohesion and friction angle on the factor of safety. These two parameters have a positive effect on the improvement of Fs while Eref remain unchanged, the slope still gains some resistance. Moreover, the cohesion is more efficient than the angle of friction to
increase $F_s$. When the friction angle is 40 kPa (i.e. the maximum value tested), the effectiveness of the cohesion on the improvement of $F_s$ is the most significant.

Figure 11b shows the interactive effects of friction angle and $E_{ref}$ on $F_s$ of the slope at a cohesion of 27.5 kPa. The additive friction angle contributes to the resistance, while the increase in $E_{ref}$ is marginal.

Figure 11. $F_s$ response surface plots for a stabilized slope with samples at a constant parameter of: (a) 7750 kPa of $E_{ref}$, (b) 27.5 kPa of cohesion and (c) 35° of friction angle.

Source: Authors
Figure 11c shows the interactive effects of cohesion and Eref on Fs of the slope at a friction angle of 30°. Similarly, the cohesion additive contributes to the strength, while the Eref increase is marginal.

In addition Figure 11b shows that the angle of friction improves Fs, while the rate of improvement is less important compared to that obtained by using cohesion (Figure 11c). However Fs is low until Eref will be 7750 kPa after an increase is marginal for Fs.

![Optimized](image)

Figure 12 shows that an interesting value is sought from the response variable can be predicted and obtained from the developed model. Thus, the cohesion of 5 kN/m², a friction angle of 30° and an Eref of 8409 kN/m², to allow a variable response (safety factor) of 1.1061 with a good desirability of 0.85990.

### 4 CONCLUSION

In this work, a methodology for optimizing the parameters of a sloping terrain using the DOE method (the Taguchi method and the RSM response surface) was applied. The goal is to optimize a mechanical model (talus) and to identify the optimal combination of soil parameters: cohesion, friction angle and Eref modulus while minimizing the safety factor. The potentials of the method to estimate the optimal parameters were explored.

We also observed that the soil modulus Eref does not affect the slope stability because the soil shear strength (stability) totally depends on the shear parameters such as cohesion and internal friction angle.
Optimization by the single-objective Taguchi method was carried out taking into account the signal-to-noise ratio (S/N). This report allowed us to conclude that the optimal values to minimize the safety factor are therefore the combination of a cohesion of 5 kN/m$^2$, an angle of friction of 30° and a modulus $E_{ref}$ of 10000 kN/m$^2$, for the RSM method $E_{ref}$ of 8409 kN/m$^2$ is obtained as an optimal value.

The RSM method (Conception Box–Behnken), is a very useful statistical method which has reduced the number of experiments. One of the advantages of the method lies in the statistical analysis which makes it possible to see the optimization problem under several facets (in particular in terms of probabilities and desirability with regard to the parameters considered). The mutual interactions between the independent variables are described with quadratic equations that predict the response under the applied conditions.

The method proposed in this study can constitute a valuable tool for optimizing the geotechnical parameters of slopes and embankments, more parameters and different geometries can be considered in future analyses.
REFERENCES


