Bearing capacity evaluation of embedded circular footing considering presence of an overload adjacent

Avaliação da capacidade de carga de sapatas circulares embutidas considerando a presença de uma sobrecarga adjacente

DOI: 10.54021/seesv5n1-098

Recebimento dos originais: 16/04/2024
Aceitação para publicação: 07/05/2024

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ABSTRACT
The bearing capacity of axially loaded embedded foundations has been widely studied using analytical and numerical methods. However, there are many discrepancies in the literature results regarding the bearing capacity factors and depth factors of circular embedded footings proposed by different authors. Terzaghi's hypothesis has been used to assess the bearing capacity of shallow foundations and several analytical solutions have been proposed for computing bearing capacity factors, even though a superposition of individual contributions may lead to some error in the computed bearing capacity. This assumption has sometimes been criticized as being more conservative. This article aims to study two problems; The first part is focuses on the numerical evaluation of the superposition hypothesis in the calculation of the bearing capacity for rough circular foundations embedded in sand. In this section, the bearing capacity factors contributing to the conventional solution have been evaluated and superposition errors are indicated and compared with those found by different authors. The latter part of the work is devoted to studying the depth effect on the estimation of the bearing capacity of a rough circular footing embedded in the sand considering presence of an overload adjacent. For both analyses, the finite-difference code
Flac2d (FLAC 2007) was used to reach the bearing capacity for embedded circular footings, the ground friction angles are between 25° and 40° and a depth ratio Df/D varies from 0.1 to 2. The calculation results are presented in tables and graphs and compared to previously published results available in the literature.

**Keywords:** numerical modelling, embedment depth, bearing capacity, superposition assumption, sand.

**RESUMO**
A capacidade de carga de fundações embutidas com carga axial tem sido amplamente estudada usando métodos analíticos e numéricos. No entanto, existem muitas discrepâncias nos resultados da literatura em relação aos fatores de capacidade portante e fatores de profundidade de sapatas embutidas circulares propostos por diferentes autores. A hipótese de Terzaghi tem sido usada para avaliar a capacidade de suporte de fundações rasas e diversas soluções analíticas têm sido propostas para calcular os fatores de capacidade de suporte, embora uma superposição de contribuições individuais possa levar a algum erro na capacidade de suporte calculada. Esta suposição às vezes foi criticada como sendo mais conservadora. Este artigo tem como objetivo estudar dois problemas; A primeira parte centra-se na avaliação numérica da hipótese de superposição no cálculo da capacidade de suporte para fundações circulares rugosas embutidas em areia. Nesta seção, os fatores de capacidade portante que contribuem para a solução convencional foram avaliados e os erros de superposição são indicados e comparados com aqueles encontrados por diferentes autores. A última parte do trabalho é dedicada ao estudo do efeito da profundidade na estimativa da capacidade de carga de uma sapata circular rugosa embutida na areia considerando a presença de uma sobrecarga adjacente. Para ambas as análises, foi utilizado o código de diferenças finitas Flac2d (FLAC 2007) para atingir a capacidade de carga para sapatas circulares embutidas, os ângulos de atrito com o solo estão entre 25° e 40° e a relação de profundidade Df/D varia de 0,1 a 2. Os resultados dos cálculos são apresentados em tabelas e gráficos e comparados com resultados publicados anteriormente e disponíveis na literatura.

**Palavras-chave:** modelagem numérica, profundidade de embutimento, capacidade de carga, suposição de superposição, areia.

**1 INTRODUCTION**

The determination of bearing capacity of shallow foundations has been the subject of much research until today, (CHOUHAN et al., 2023; JANABI et al., 2023; KHATRI; YADAV; SHRIVASTAVA, 2022; ZHAI et al., 2023). (TERZAGHI, 1943) expressed his first bearing capacity equation for a strip footing as:

\[ q_u = cN_c + q_0N_q + \frac{1}{2}\gamma B N_f \]  

(1)
Where:

\[ c: \text{cohesion, } q: \text{surcharge, } \gamma: \text{unit weight of soil, } B: \text{width of footing, and } N_c, N_q \text{ and } N_c \]

are the bearing capacity factors, which are functions of the friction angle \( \varphi \).

The ultimate bearing capacity is estimated by superposition of three basic components. The first term in Eqs. (1), reflects the contribution of cohesion to the ultimate bearing capacity, and the second term reflects the frictional contribution of the overburden pressure or surcharge. The last term reflects the frictional contribution of the self-weight of the soil in the failure zone. Terzaghi assumed the soil to be a semi-infinite, isotropic, homogeneous, weightless, rigid plastic material, the footing to be rigid, and the base of the footing to be sufficiently rough to ensure there is no separation between the footing and the underlying soil. It also was assumed that the failure occurs in the general shear mode. (TERZAGHI, 1943) and (HANSEN, 1970) neglected the shear resistance provided by the overburden soil (neglect shear along cd), which was treated as a surcharge (i.e., above the footing level), which was included in the modifications made by Meyerhof that are discussed here. (MEYERHOF, 1951) obtained, with a similar technique of the Terzaghi’s approach, approximate solutions to the plastic equilibrium of shallow foundations and deep foundations, assuming a different failure mechanism, the influence of the shear strength of soil above the base of the foundation was investigated. (MEYERHOF, 1951) suggested a general bearing capacity theory with consideration for correction factors for eccentricity, load inclination, foundation depth. To take into account, the effects of embedment and shape footing, the conventional bearing capacity Eqs. (1) is modified by the correction factors. The bearing capacity formula can be written as:

\[
q_u = s_c d_c c N_c + s_q d_q q_0 N_q + \frac{1}{2} s_\gamma d_\gamma \gamma BN_\gamma
\]

(2)

Where \( s_c, s_q \) and \( s_\gamma \) are shape factors and \( d_c, d_q \) and \( d_\gamma \) are depth factors. The principal authors having proposed these factors are (HANSEN, 1970; MEYERHOF, 1963; TERZAGHI, 1943; VESIC, 1973). From Eqs. (2), for an embedded circular footing under vertical load on cohesionless soil, this means that this equation can be reduced to:
\[ q_u = s_d q_0 N_q + \frac{1}{2} s_r \gamma B N_r \quad (3) \]

For many years the concept of superposition has been used to assess the bearing capacity of shallow foundations. Several analytical solutions have been proposed for computing bearing capacity factors. Even though a superposition of individual contributions may lead to some error in the computed bearing capacity, this method has been widely used for its simplicity. Thus, the assumption of superposition turns out to be an approximation, without precision, but reliable. According to (DAVIS; J. R. BOOKER, 1971), the error due to adoption of superposition hypothesis may be over 20%. (BOLTON; C. K. LAU, 1993) corroborated that in the safety margin, the superposition hypothesis turned out to be systematically inaccurate, with a margin exceeding 20%. From the results found by (MICHALOWSKI, 1997) the load capacity factor caused by the weight of the soil increased in proportion to the increase in the ratio \( q/\gamma B \) and \( c/\gamma B \). Using characteristics method for rough strip footing, (SMITH, 2005) found that the error caused by superposition assumption 25% approach.

The bearing capacity of an embedded footing is greater than that of a footing on the surface. This is explained by the fact that additional work is generated to move the mass of sand in the passive zone towards the existing overload above of footing, and also in the condition of mobilization of additional shear resistance along the sliding surface portion above the base of footing.

Previous research has applied a direct approach in calculating the bearing capacity of embedded footing, without superimposing the three bearing capacity terms. In this context, the determination of the bearing capacity of shallow foundations has frequently been carried out. (LYAMIN et al., 2007) and (KHATRI; YADAV; SHRIVASTAVA, 2022) using the finite-element method, addressed numerical investigations of the bearing capacity of embedded footings on cohesionless soil, and presented values of depth factors for use in bearing capacity computations in sand. The superposition of the effects of the overload and the weight of the structure is not exact from a theoretical point of view. In reality, the additional load results from the gravitational force acting on the ground above the base of the foundation. (LYAMIN et al., 2007), found that superposition
assumptions are not valid. Using two-dimensional upper bound numerical limit analysis, (ANTÃO; SILVA; GUERRA, 2010) proposed a depth factor of shallow foundations in sand. (JANABI et al., 2023) carried out a series of penetration tests on models of strip and square footings in sand, at the surface and for different depth ratios, which allows to obtain experimentally shape and depths factors.

Moreover, we can register notable divergences in the estimation of the depth factors, for example, according to (LYAMIN et al., 2007), the values of $d_q$ for several shapes of embedded footing on sand, decrease with the increase in the $D_f/D$ ratio and does not approach 1 when the footing is on the surface, they explained this result by the non-logic of the superposition hypothesis. Contrary to literature for strip footing, (JANABI et al., 2023) found that $d_q$ decrease with the increase in the $D_f/D$ ratio. While for the factor $d_y$ is taken equal to 1. i.e., the effect of sand shear resistance along the part of the sliding surface above the base of the footing is assumed to be fully captured by $d_q$ factor.

From previous works, it is found that the representation of the ground above the level of the footing as an equivalent overload elicits a response distinct from that of the ground below the base of the footing, with respect to the interaction between soils. Consequently, this hypothesis sometimes has been criticized as being over conservative. For this reason, this work aims to carry out a numerical study in Flac2d (FLAC 2007), which is interested in studying the validity of the superposition theory and evaluating its impact on the calculation of the bearing capacity of circular surface foundations embedded in the sand, as well as the estimation of the depth effect in the event of the presence of an overload adjacent to a circular embedded footing on the evaluation of the depth factors of bearing capacity $d_y$ and $d_q$, for a circular footing embedded on the sand.

2 PROBLEM DEFINITION

This study aims to analyze two problems; the first concerns the validation of the superposition assumption for a rough circular embedded footing. In this part, the bearing capacity is determined for three different cases, the first case is presented in form of two models illustrating the hypothesis of superposition. The first model is that of a circular footing on the surface of weightless soil and the second is of a circular footing on the surface of non-heavy soil; with the presence
of an adjacent overload, which replaces the soil above base of footing (Fig. 1.a). For the second case, the estimation of the bearing capacity is for a circular footing on the surface of weightless soil with the presence of equivalent surcharge used to replace the soil above base of footing ($q_{u2}$), (i.e., direct calculation $q_{u2}$) (Fig. 1.a). The last case illustrates a circular footing which modelled as an embedded footing ($q_{u3}$), as shown in Fig. 1.b. The second part of the work concerns the study of the effect depth on the estimation of the bearing capacity of a rough circular footing in the sand.
3 NUMERICAL MODELING

The problem studied considers circular footings embedded in cohesionless soil, under vertical load. In this paper, the finite-difference code Flac2d (FLAC 2007) was used to reach the bearing capacity for embedded circular footings. The base of the foundation was modelled as fully rough with smooth sides. In the current modeling study, the diameter $D$ of the footing is 2m and it is embedded at a variable depth $D_f$. The depth ratio $D_f/D$ of 0 (surface), 0.1, 0.2, 0.4, 0.6, 0.8, 1, and 2 were considered. Since the problem is axis-symmetric, only half of the problem domain is considered.

Before starting the numerical simulation part, it is obvious to carry out a preliminary study which concerns the comparison of numerical results of bearing capacity factor $N'_q$, obtained for three different vertical cross-sectional shapes of circular shallow foundation, with friction angle values 25° and 30°. The models rectangular shape (block), T shape and rigid plate vertical cross-sections, were studied (shown in Fig. 2). A comparison of previous test results, shown that the bearing capacity factor $N'_q$ of foundations on the three models taking totally similar with a neglected difference (Fig. 3).
After the preliminary simulations, all analyses were performed by applying uniform vertical displacements and zero horizontal displacements to the nodes along the boundary ABC until failure was reached Fig. 4. Fig. 5 shows the mesh retained for this analysis.
The elastic perfectly plastic Mohr Coulomb model encoded in Flac2d is used. Physical and mechanical characteristics used in the present study are a shear modulus $G = 10$ MPa, an elastic bulk modulus $K = 20$ MPa and a soil unit weight $\gamma = 20$ KN/m$^3$. A series of four values of the angle of soil internal friction $\varphi = 25–40^\circ$ with an increment of $5^\circ$ and for an associated soil ($\psi = \varphi$) is considered.

The loading of footings is simulated by imposing equal vertical velocities, to the nodes of the footing surface represented by the boundary AB (Fig. 4) until failure. After running a number of verifications, the magnitude of vertical velocities is finally chosen to be $1 \times 10^{-7}$ m/s downward, which is small enough to minimize the influence of initial velocity on the results.

Using a FISH function, the ultimate bearing capacity $q_u$ was calculated by dividing the computed ultimate total vertical load by the area of the footing. It was pointed out that the load was calculated as the sum of the vertical reaction forces along the footing level ‘AB’ as expressed by Eqs. (4). This load includes the resistance from both the base and the vertical sides of the footing.

\[
q_u = \frac{2 \sum f_i r_i}{r^2} \tag{4}
\]

Where:

$q_u$ is the ultimate bearing capacity; $f_i$ is the reaction force in the vertical direction at footing grid point $i$; $r_i$ is the associated radius at grid point $i$; $r$ radius of the footing.

For cohesionless soil, the ultimate bearing capacity equation has the following form:
Where:

\( N'_q \) and \( N'_\gamma \) are the bearing capacity factors corresponding to circular with \( D_f \) depth.

For \( N'_q \) determination, it is assumed that the soil has neither a unit weight nor a cohesion (i.e., \( \gamma = 0, c = 0 \)), but a uniform surcharge is present over the ground surface, which represent the weight of the soil adjacent to the footing base at the base level. The same value of overload is also applied on the footing itself. This is simplified as follows:

\[
N'_q = \frac{q_u}{q_0}
\]

(6)

The factor \( N'_\gamma \) can be assessed simply by taking the soil as non-cohesive (i.e., \( c = 0 \)) and with no surcharge (i.e., \( q = 0 \)) on the ground surface. This simplification gives:

\[
N'_\gamma = \frac{2q_u}{\gamma B}
\]

(7)

4 RESULTS AND DISCUSSIONS

4.1 ASSESSMENT OF SUPERPOSITION

Before computing the bearing capacity factors \((N'_q \text{ and } N'_\gamma)\) of embedded rough circular footing on sand, the superposition assumption in the calculation of the ultimate bearing capacity \(q_u\) was treated for soil friction angles range from 25° to 40°, depth ratio \(D_f/D\) varied from 0.1 to 2 and compared with existing solutions available in the literature (Table 1). The obtained values of \(q_u\) were compared with the analytical results given by (HANSEN, 1970; MEYERHOF, 1963; TERZAGHI, 1943). Table 1, shows the comparison of bearing capacity values of a circular footing obtained by different authors. Knowing that \( q_{u1} \) represents the bearing capacity, determined by the principle of superposition, this is the sum of the two tests \( q_{u11} \) and \( q_{u12} \) mentioned above. The bearing capacity \( q_{u2} \) is estimated by the
direct calculation; the case of circular footing on the surface of soil weighing with the presence of an equivalent surcharge used to replace soil above base of footing, \( q_{u3} \): represents the bearing capacity assessed for the case of an embedded circular footing and shown graphically in Fig. 6. It can be seen from Table 1 and Fig. 6, that values of the bearing capacity increase considerably with increasing \( D_f/D \) and \( \varphi \). It can also be noted that the values obtained by Flac2d are considerably greater than to those found by other authors where the superposition hypothesis is assumed. For the first two cases, the depth of footing is replaced by an overload, but the highest values of the bearing capacity are given for the case where direct calculation is used \( (q_{u2}) \). The error rate is between 4.97% and 14.14%, for soil friction angles from 25° to 40° and the values for both cases are very close for a high depth ratio. Referring to Fig. 7, it is observed that the error rate between \( q_{u2} \) and \( q_{u3} \) increases considerably with the increases of depth ratio and is almost constant with the variation of the angle of friction. The margin of error reaches 48% for a depth ratio \( D_f/D = 2 \). Moreover, the analytical results obtained by Meyerhof are very close to the numerical ones for the case of superposition \( (q_{u1}) \), especially for low soil friction angles.

| Table 1 - Bearing capacity of a circular footing for different cases and different authors |
|---------------------------------|---------|---------|---------|---------|---------|---------|
| \( \varphi \) | \( D_f/D \) | \( q_{u1} \) | \( q_{u2} \) | \( q_{u3} \) | Terzaghi 1943 | Meyerhof 1963 | Hansen 1970 |
| 25°   | 0.1   | 204.44 | 238.10 | 260.40 | 150.96 | 225.39 | 143.90 |
|       | 0.2   | 281.20 | 333.10 | 374.00 | 201.84 | 283.69 | 210.47 |
|       | 0.4   | 430.70 | 494.80 | 612.50 | 303.60 | 405.29 | 354.96 |
|       | 0.6   | 580.70 | 651.30 | 867.10 | 405.36 | 533.56 | 514.60 |
|       | 0.8   | 728.90 | 806.00 | 1149.00 | 507.12 | 668.51 | 689.38 |
|       | 1     | 878.80 | 959.30 | 1448.00 | 608.88 | 810.14 | 879.31 |
| 30°   |       | 1     | 1623.50 | 1716.00 | 2325.00 | 1117.68 | 1618.36 | 1718.06 |
|       |       | 2     | 1868.60 | 2028.00 | 3067.00 | 1127.96 | 1600.51 | 1603.54 |
|       | 35°   | 0.1   | 1268.90 | 1385.00 | 1514.00 | 710.68 | 1222.58 | 621.98 |
|       | 0.2   | 1600.90 | 1821.00 | 2042.00 | 876.44 | 1434.98 | 847.59 |
Figure 6. Variation of bearing capacity with $D_f/D$ and $\varphi$ for a rough circular footing (a) $\varphi=25^\circ$, (b) $\varphi=40^\circ$.

Source: Authors

Figure 7. Variation of Error rate between $q_{u2}$ and $q_{u3}$ with $\varphi$ for a rough circular footing

Source: Authors
4.2 EVALUATION OF ERROR MARGINS OF THE SUPERPOSITION HYPOTHESIS

Fig. 8, compare the results acquired by Flac2d for the third case (embedded circular footing), with those found by several authors cited in the previous section, it was concluded that the values obtained by literature, where the superposition assumption is adopted are considerably little than those obtained by Flac2d. The bearing capacity is founded for different depth ratio and for diverse friction angles. The Terzaghi’s values were compared with those calculated directly; the error rate is between 42.02% and 78.23%. The error rate increases with increasing depth ratio and friction angle. While the error rate noted by Meyerhof’s results was between 13.44 % and 59.63%. The effect of the friction angle is not very marked. Hansen’s results have led to a high error rate ranging from 39.27% and 68.55%. These error rates are almost independent of the depth ratio. It can be seen from the curve of Fig. 8, that the error rate between the results of the third case ($q_{u3}$) and those of Hansen, forms a peak when the ratio changes from 1 to 2.

![Figure 8. Error rate between Flac2d ($q_{u3}$) and the literature: (a) $\phi = 25^\circ$, (b) $\phi = 40^\circ$](source: Authors)

4.3 FAILURE MECHANISMS

Examples of failure mechanisms obtained by a numerical modeling of a rough circular footing with Flac2d are shown in Fig. 9. For the two cases studied in the previous section (i.e., $q_{u2}$ is the bearing capacity is calculated by the direct calculation and $q_{u3}$ is the bearing capacity assessed for the case of an embedded circular footing) for case friction angle of the soil 30° and the depth ratio $D_f/D= 0.1, 0.6$ and 1. It can be seen from the next figure that the size of the shear zone of the rupture mechanism in the case of an embedded circular footing is greater
than that of the first case. Moreover, for the second case, it is clear that the area of rupture extends above the base level of footing. Whereas in the Terzaghi analysis, the shear strength of the soil above the base level of footing is neglected, which explains the error due to the superposition hypothesis, this error increases with increasing the depth of footing.
Figure 9. Contours of maximum shear strain for rough circular footing in the case $\phi=30^\circ$ (a) equivalent surcharge used to replace soil above base of footing; (b) footing modelled as an embedded footing.

Based on the comparison obtained by Flac$^{2d}$ results and others available in literature, for bearing capacity of an embedded circular footing with a depth ratio
ranging from 0.1 at 2 and a friction angle varies between 25° to 40°. It is obvious that the present study proves that the bearing capacity values given by the superposition method are very smaller than those of Flac$^{2d}$, where footing is modelled as an embedded footing. The superposition always underestimates the bearing capacity. Although this assumption always errs on the safe but the errors invoked by it use exceed 50%.

4.4 EVALUATION OF BEARING CAPACITY FACTORS $N'_\gamma$ AND $N'_q$

The numerical modeling procedure was first evaluated for $N'_\gamma$ and $N'_q$ for surface circular footing in sands. From Eqs. (3), the bearing capacity factors are $N'_\gamma = 2q_u/\gamma D$, when $c = 0, q = 0$ and $N'_q = q_u/q$ when $\gamma = c = 0$ for rough footing and soil friction angles range from 25° to 40°. Table 2 listed values of $N'_\gamma$ as a function of $\varphi$ from the finite difference method together with those from (BOLTON; C. K. LAU, 1993; ERICKSON; ANDREW DRESCHER, 2002; KUMAR; MANASH CHAKRABORTY, 2015; LOUKIDIS; R. SALGADO, 2009; MANOHARAN; S. P. DASGUPTA, 1995; MARTIN, 2004; MEYERHOF, 1963; TAGHVAMANESH; R. ZIAIE MOAYED, 2021; TERZAGHI, 1943). It can be seen that the present study $N'_\gamma$ values are larger those reported by Terzaghi, while the results obtained by (ERICKSON; ANDREW DRESCHER, 2002; KUMAR; MANASH CHAKRABORTY, 2015; LOUKIDIS; R. SALGADO, 2009; MANOHARAN; S. P. DASGUPTA, 1995; MARTIN, 2004; MEYERHOF, 1963; TAGHVAMANESH; R. ZIAIE MOAYED, 2021) are very close to the values of $N'_\gamma$ obtained by the present computations. The results of (BOLTON; C. K. LAU, 1993) appear to be very high.

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Source: Authors

Table 3, illustrated the comparison of bearing capacity factor $N'_q$ for rough
circular footing. The results of present solution are agreed with the result obtained by (LOUKIDIS; R. SALGADO, 2009; MANOHARAN; S. P. DASGUPTA, 1995; MARTIN, 2004). Whereas values of (BOLTON; C. K. LAU, 1993; MEYERHOF, 1963; TERZAGHI, 1943) are found to be lower than the values obtained in the present analysis.

Table 3 - Comparison of $N'_q$ values from this study with the results derived by other authors for rough surface circular footings

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Source: Authors

Several numerical modeling has been carried out to study the effect of embedment ratio $D_f/D$ on bearing capacity factors ($N'_f$ and $N'_q$) for a rough circular footing and for different values of angle of soil internal friction $\varphi$. The computation results as defined in Eqs. (6) and (7) for $N'_f$ and $N'_q$ respectively are given in Tables 4 and 5. The results of (LYAMIN et al., 2007), were also included in Table 6 for the purpose of comparison.

Table 4 - Variation of the bearing capacity factor $N'_f$ with $D_f/D$ and $\varphi$

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</tbody>
</table>

Source: Authors

For more illustration, Fig. 10 provides a comparison of the present results of $N'_f$ with results of (LYAMIN et al., 2007). A good agreement is observed between the present results of $N'_f$ and the results of (LYAMIN et al., 2007). The $N'_f$ of embedded circular footing increases with increasing of ratio $D_f/D$ and of $\varphi$. 
Figure 10. The variation of $N'_q$ with $D_f/D$ for $\varphi$ ranging from 25° to 40°

Table 5 summarized the values of $N'_q$ of an embedded circular footing with a depth ratio which varies between 0.1 and 2. The soil is assigned a friction angle ranging from 25° to 40° and an adjacent overload to the footing ($q_0=10$KPa) as shown in Fig. 11. In the case of the presence of an adjacent overload near an embedded circular footing, it can be seen from Table 5 that $N'_q$ the factor of the bearing capacity increased with the increase of the depth ratio. The depth effect is significant (significantly large) for high ratio, the factor augmented from 199 to 495.3 respectively for the case of the footing on the surface and for an embedded circular footing with a depth ratio $D_f/D=2$ with a friction angle $\varphi=40°$. In addition, as shown in Fig. 12, the factor $N'_q$ depends clearly on the soil friction angle, its values raise with the raising of the angle, this increase is very noticeable for high angles.

<table>
<thead>
<tr>
<th>$D_f/D$</th>
<th>$\varphi = 25°$</th>
<th>$\varphi = 30°$</th>
<th>$\varphi = 35°$</th>
<th>$\varphi = 40°$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>18.93</td>
<td>38.39</td>
<td>84.02</td>
<td>199.00</td>
</tr>
<tr>
<td>0.1</td>
<td>21.28</td>
<td>43.18</td>
<td>94.42</td>
<td>221.50</td>
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<tr>
<td>0.2</td>
<td>23.13</td>
<td>46.92</td>
<td>102.20</td>
<td>238.80</td>
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<tr>
<td>0.4</td>
<td>26.13</td>
<td>53.77</td>
<td>116.20</td>
<td>270.20</td>
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<tr>
<td>0.6</td>
<td>29.18</td>
<td>59.82</td>
<td>129.50</td>
<td>299.80</td>
</tr>
<tr>
<td>0.8</td>
<td>31.99</td>
<td>65.92</td>
<td>142.40</td>
<td>328.70</td>
</tr>
<tr>
<td>1.0</td>
<td>34.82</td>
<td>71.84</td>
<td>155.30</td>
<td>357.10</td>
</tr>
<tr>
<td>2.0</td>
<td>47.76</td>
<td>100.00</td>
<td>217.00</td>
<td>495.30</td>
</tr>
</tbody>
</table>

Source: Authors
4.5 SCALE EFFECT

To study the effect of footing scale on the assessment of the bearing capacity factors, a numerical tests were conducted on embedded circular footings for the same depth ratio $D_f/D$ equal to 0.5, 1 and 2 and for an friction angle $\varphi =25^\circ$. Analysis was repeated for different values of width and depth. According to the results of the Table 6, it can be observed that the factors $N'_\gamma$ and $N'_q$ are constant for the same depth ratio of the footing, so that the adimensionality of these factors can be confirmed in this case.

Table 6 - Bearing capacity factor with a different geometry for $\varphi =25^\circ$.

<table>
<thead>
<tr>
<th>$D_f/D$</th>
<th>$D_f$</th>
<th>$D$</th>
<th>$N'_\gamma$</th>
<th>$N'_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.4</td>
<td>36.60</td>
<td>27.69</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.8</td>
<td>36.30</td>
<td>27.68</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>36.70</td>
<td>27.63</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>70.19</td>
<td>34.79</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>70.61</td>
<td>34.82</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>0.2</td>
<td>155.80</td>
<td>47.77</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>0.4</td>
<td>156.10</td>
<td>47.80</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>156.50</td>
<td>47.69</td>
</tr>
</tbody>
</table>

Source: Authors
4.6 DEPTH FACTORS $d_\gamma'$ AND $d_q'$

In this section, we present results of numerical modeling adopted to obtain values of depth factors $d_\gamma'$ and $d_q'$ for employ in bearing capacity calculation of an embedded rough circular footing. The depth factors are determined by calculating the bearing capacity for several depth ratio and compared with that calculated in the case of a circular footing located on the ground surface. The soil friction angles used in this part varies between 25° and 40° with a step of 5°. Firstly, the $d_\gamma'$ factor is determined for the case of soil weighing ($\gamma \neq 0, q = 0$) according to Eqs. (8).

$$
\frac{d_\gamma'}{\gamma} = \frac{q_u}{D_f \neq 0} \frac{\gamma_c}{q_u |D_f = 0}
$$

(8)

It can be seen from Fig. 13 that values of $d_\gamma'$ increases gradually as value of $D_f / D$ rises, while depth factor of this study decreases with increases friction angle. It can also note that the values of Flac 2D are very close to those of (LYAMIN et al., 2007). On the other hand, it is note that for most previous work, classic hypothesis is to take the value of equal to 1. that is, the effect of sand shear resistance along the part of the sliding surface above the base of an embedded footing is assumed to be fully captured by the depth factor $d_q'$ (JANABI et al., 2023).

Eqs. (9) gives the values of depth factor $d_q'$ for embedded circular footing

![Figure 13. Variation of the depth factor $d_\gamma'$ with $D_f / D$ and $\varphi$](source: Authors)
with the presence of an overload adjacent to the ground surface.

\[ d'_q = \frac{q_u|_{D_f=0}}{q_u|_{D=0}} \]  \hspace{1cm} (9)

Table 7, illustrates the variation of the depth factor \(d'_q\) with the depth ratio and the friction angle of soil. According to Fig. 14, it is clear that the depth factor \(d'_q\) increase with the increase of the depth ratio \(D_f/D\), it also evident that the depth factor \(d'_q\) is independent from the angle friction. The curves mentioned in Fig. 14 can be used to establish the trend curve shown in Fig. 15. This curve allows us to obtain a mathematical formula for the determination of the depth factor \(d'_q\). The equation can be written in the following form:

\[ d'_q = 0.764 \left( \frac{D_f}{D} \right) + 1.0539 \]  \hspace{1cm} (10)

<table>
<thead>
<tr>
<th>(D_f/D)</th>
<th>(\varphi = 25^\circ)</th>
<th>(\varphi = 30^\circ)</th>
<th>(\varphi = 35^\circ)</th>
<th>(\varphi = 40^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.1</td>
<td>1.12</td>
<td>1.12</td>
<td>1.12</td>
<td>1.11</td>
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<tr>
<td>0.2</td>
<td>1.22</td>
<td>1.22</td>
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<td>1.20</td>
</tr>
<tr>
<td>0.4</td>
<td>1.38</td>
<td>1.40</td>
<td>1.38</td>
<td>1.36</td>
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<td>1.54</td>
<td>1.51</td>
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<tr>
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<td>1.72</td>
<td>1.69</td>
<td>1.65</td>
</tr>
<tr>
<td>1</td>
<td>1.84</td>
<td>1.87</td>
<td>1.85</td>
<td>1.79</td>
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<tr>
<td>2</td>
<td>2.52</td>
<td>2.60</td>
<td>2.58</td>
<td>2.49</td>
</tr>
</tbody>
</table>

Source: Authors
Figure 14. Variation of the depth factor $d'_q$ with $D_f/D$ and $\varphi$

![Graph showing variation of depth factor $d'_q$ with $D_f/D$ and $\varphi$.]

Source: Authors

Figure 15. The trend curve of variation of depth factor $d'_q$

![Graph showing trend curve of variation of depth factor $d'_q$.]

Source: Authors

5 CONCLUSIONS

The finite-difference code Flac2d was used to discuss the validity of Terzaghi's superposition hypothesis and study the depth effect on the assessment bearing capacity factors for a rough circular embedded footing. The foundation is subjected to an axial load and resting on a ground characterized by the Mohr-Coulomb yield criterion and associative flow rule. The obtained results are presented in the form of graphs and tables. From the comparisons with various studies available in the literature, the following conclusions can be drawn:

- It is obvious that the present study proves that the bearing capacity values given by the superposition method are very smaller than those of Flac$^{2d}$, where direct calculation is adopted. The superposition always underestimates the bearing capacity. Although this assumption always errs on the safe but the errors invoked by it use exceed 50%. It would be fair to
say that this assumption is not valid in the case of an embedded circular footing in sand.

- The $N'_γ$ results from the present work are in good agreement with the solutions obtained by (LYAMIN et al., 2007). The $N'_γ$ of embedded circular footing increases with increasing of ratio $D_f/D$ and of $φ$.

- In the case of presence of an adjacent overload near an embedded circular footing $N'_q$ increases with the increase of the depth ratio, the depth effect is significantly large for high depth ratios $D_f/D$. It should also be noted that $N'_q$ depends clearly on the soil friction angle, its values raise with the raising of the angle, this increase is very remarkable for high angles.

- From this study, it would be fair to say that $N'_γ$ and $N'_q$ are constant for the same depth ratio of the footing, so that the adimensionality of these factors can be confirmed in this case.

- The $d'_γ$ values are proportional to the depth ratio $D_f/D$ and decrease with the increase of the friction angle.

- A simplified equation can be proposed from curves to determine the values of $d'_q$ factor. It would appear that the depth factor increases with increasing depth ratio $D_f/D$ and it is independent of friction angle.

The results of this research are essential to guarantee the efficiency of built infrastructures while contributing to the advancement of knowledge in the field of engineering and geotechnics. Their economic importance lies in reducing construction costs through optimized foundation design, resulting in significant savings for engineering projects.

Future studies aimed at improving this work:

- Conduct experimental studies to corroborate the results obtained and extend the research to various forms of foundations, types of soil (heterogeneous soil) and loading cases.
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