FEM analysis of thermomechanical effects on stress intensity factors

Análise FEM dos efeitos termomecânicos nos fatores de intensidade de tensão

DOI: 10.54021/seesv5n1-106

Recebimento dos originais: 19/04/2024
Aceitação para publicação: 10/05/2024

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ABSTRACT
The response of a material to thermo-mechanical loading depends on its thermal and mechanical properties. Some materials may exhibit significant thermal expansion, meaning they expand significantly when heated, while others may be stiffer and show little thermal expansion. To evaluate the behavior of a material or structure subjected to thermo-mechanical loading, advanced analysis and modeling techniques are used, such as finite element modeling. These methods make it possible to predict deformations, stresses and possible failure problems that may occur under the effect of thermo-mechanical loading. In the present paper, assuming that the thermal regime was steady, the effects of thermo-mechanical loads on the stress intensity factor in a 2D cracked plate of functional gradient material (FGM) are examined. The analyzes the effects of thermo-mechanical loads on the stress intensity factor in a 2D cracked plate of functionally
gradient material is a complex problem and generally requires advanced modeling and simulation approaches, such as the finite element method. The plate has a crack on the edge and is made of FGM (titanium-zirconia). Analyses are performed for different imposed temperature values, using the Newman’s conditions. The problem is solved using a newly created USDFLD subroutine in the ABAQUS program. In this routine, functionally graded material property variations follow an exponential law function in the plate with cracks. Stress intensity factors are calculated using the J-integral, taking into consideration the variation in properties at the crack tip. Effects of temperature and relative crack length on stress intensity factors were evaluated. The stress intensity factor values obtained by the finite element method (FEM) modeling shows that there is a good correlation with the results obtained in other studies.

**Keywords:** thermo-mechanical, stress intensity factor, functional gradient material, FGM.

**RESUMO**
A resposta de um material à carga termomecânica depende de suas propriedades térmicas e mecânicas. Alguns materiais podem apresentar uma expansão térmica significativa, o que significa que eles se expandem significativamente quando aquecidos, enquanto outros podem ser mais rígidos e apresentar pouca expansão térmica. Para avaliar o comportamento de um material ou estrutura sujeito a cargas termomecânicas, são usadas técnicas avançadas de análise e modelagem, como a modelagem de elementos finitos. Esses métodos possibilitam a previsão de deformações, tensões e possíveis problemas de falha que podem ocorrer sob o efeito de cargas termomecânicas. No presente trabalho, supondo que o regime térmico seja estável, são examinados os efeitos das cargas termomecânicas sobre o fator de intensidade de tensão em uma placa rachada 2D de material de gradiente funcional (FGM). A análise dos efeitos das cargas termomecânicas sobre o fator de intensidade de tensão em uma placa rachada 2D de material com gradiente funcional é um problema complexo e geralmente requer abordagens avançadas de modelagem e simulação, como o método de elementos finitos. A placa tem uma rachadura na borda e é feita de FGM (titânio-zircônia). As análises são realizadas para diferentes valores de temperatura impostos, usando as condições de Newman. O problema é resolvido usando uma subrotina USDFLD recém-criada no programa ABAQUS. Nessa rotina, as variações de propriedade do material com graduação funcional seguem uma função de lei exponencial na placa com rachaduras. Os fatores de intensidade de tensão são calculados usando o integral J, levando em consideração a variação das propriedades na ponta da trinca. Os efeitos da temperatura e do comprimento relativo da rachadura nos fatores de intensidade de tensão foram avaliados. Os valores do fator de intensidade de tensão obtidos pela modelagem do método de elementos finitos (FEM) mostram que há uma boa correlação com os resultados obtidos em outros estudos.

**Palavras-chave:** termomecânica, fator de intensidade de tensão, material de gradiente funcional, FGM.
1 INTRODUCTION

Metals and other conventional materials have long been utilized extensively in manufacturing. But in many industries, including aerospace, submarine, defense, and nuclear, where they are used in increasingly harsh conditions related to temperature, mechanical load, and temperature, these materials' mechanical qualities fall short of the specific requirements. Due to their intrinsic versatility and ability to modify their mechanical properties along the thickness of plates and beams, functional gradient materials (FGMs) are the ideal choice to meet these needs (Shukla A. 2006). In this paper a numerical investigation of the thermo-mechanical effects on the stress intensity factor in FGM is presented.

However, the various constituent materials of the composites do not necessarily have identical thermal and mechanical properties (Density, Elasticity modulus, Poisson coefficient, etc.). Spatial variation of properties strongly influences the load response. The differential expansion generated causes the occurrence of manufacturing defects which provoke residual stresses at the interfaces of the various composite components and this may causes the tensions linked to discontinuities at bi-material interfaces to relax (Youngblood et al. 1978, Lee et al. 1994, Ravichandran 1995, Noda 1999, Nomura et al. 2001). These defects decrease material's mechanical strength and, therefore, promote delamination and can lead to sudden ruptures and catastrophic situations. In FGMs of ceramic-metallic type, the cracks are generated in several ways depending on the conditions of variation of the graded properties and the applied loading, in the case of the above materials subjected to thermal shocks or thermal fatigue (Kawasaki et al. 1996, Kawasaki et al. 2002, Rangaraj et al. 2003, Bahr et al. 2003, Iqbal et al. 2022). In addition to the fracture behavior of FGMs, other studies on FGMs include the effect of a non-uniform temperature field on the behavior of FG-CNT structures (Bhagat et al. 2020) and the analysis of forced or damped vibrations of functionally graded beams (Heydari 2022, Elmeiche et al. 2022). Crack initiation can also occur along the interface of a multilayer FGM (Tohgo et al. 2006).

Users and producers of aircraft and automobiles work tirelessly to avoid service interruptions. Since fracture continues to be the predominant cause of FGM failure, understanding how these materials behave mechanically during
fracture is essential (Erdogan 1995). The majority of these investigations on FGM failure have concentrated on linear elastic behavior. (Anlas et al. 2000) examine SIFs in FGMs for edge-cracked plate under uniform mechanical loads using both the strain energy and the J-integral. (Menouillard et al. 2006) extended the interaction integral technique approach to forecast stress intensity parameters. Using the complex variable method, (Naved et al. 2021) developed analytical formulations for determining the SIFs for cracks in the form of arcs of a circle.

The major drawback of the two previous approaches being the difficulty in separating the values of $K_I$ and $K_{II}$, in the case of mixed modes. For this last method, the meshes are more refined for an excellent numerical representation of the stress fields near the tip of crack. In order to facilitate evaluation of stress intensity factors, singular finite elements are generally created in commercial computer codes, the calculation of the stress intensity factors is done directly from the nodal displacements without the use of subroutines. To determine the stress intensity parameters for interface cracks between various anisotropic materials under mechanical stresses, a technique known as virtual crack growth is used (Parks 1974).

(Kim et al. 2002, Kim et al. 2003) used the Modified Crack Closure Method, the Mixed-Mode J-integral, the Interaction Integral, and other finite element techniques based on the theories of fracture mechanics.

The Equivalent Domain Integral (EDI) technique performed by (Gu et al. 1999) is used to determine stress intensity parameters in functionally graded materials. In a graded orthotropic band with finite width and static loading, mode I cracking problems are taken into account (Guo et al. 2004). An extension of isogeometric analysis with orthotropic technique has been proposed to numerically model stationary cracks in FGMs (Gu et al. 1999). Additionally, the fracture behavior in FGMs has been investigated using ABAQUS software (Guo et al. 2004).

For isotropic plate FGMs subjected to thermal and mechanical loading conditions, the stress intensity factors (SIF) are computed using a displacement extrapolation (DET) technique (Shojaee et al. 2015). The same author (Martinez et al. 2015) employed a different method based on generalized displacement
correlation (GDC) to establish SIFs for isotropic plate FGMs under mechanical and thermal loads. Through the use of parameters that are set at the center of gravity of each element, this technique integrates continual fluctuations in material properties. Analyses of crack propagation in FGMs under mixed-mode loading subjected to mechanical loads using the strain energy density (SED) approach and the finite element method (FEM) are performed (Benamara et al. 2017). (Dag et al. 2009) have suggested a brand-new computation technique based on the equivalent domain integral for the investigation of Mode-I fractures in thermally stressed orthotropic FGMs. A thermo mechanical analysis of an FGM subjected to thermal shock containing cracks at the edge has been developed (Mete et al. 2021). This approach is based on a new numerical technique that considers the temperature in real-time of the dependent material properties. Several authors (Jamal-Omidi et al. 2023, Paarmann et al. 2020) have modeled the thermal stresses using a decoupled quasi-static thermo elasticity model. The evaluation of transient thermal SIF was made using the 3D-FEmethod in ABAQUS and singular elements in the crack tip (Nabavi et al. 2020, Dassault 2014).

2 J-INTEGRAL FOR FGM

In isotropic FGMs, the J-integral approach can be utilized to calculate the stress intensity components. The change in potential energy of a cracked structure brought on by extension between two crack lengths is known as the J-integral in the fracture mechanics and can be expressed as follows:

\[ J = \lim_{\Gamma \to 0} \int_{\Gamma} (W \delta_{ij} - \sigma_{ij} u_j) n_i dC \]  \hspace{1cm} (1)

The arbitrary closed contour, which starts on the underside of the crack and ends on its upper side, is called \( \Gamma \). The displacement field and the stress tensor are denoted by \( u_j \) and \( \sigma_{ij} \) and \( n_i \) is the unit vector perpendicular to the closed path, \( W \) is the strain energy density. The values of this integral are often determined through the finite element approach.

Using numerical integration, \( J \) is computed. This method's benefit is associated with elasticity and plasticity evaluations. The stress intensity factors for a homogeneous material are obtained by determining the integral quantity \( J \), which
is provided by the following relationship:

$$K = \sqrt{EJ/(1-v^2)}$$  \hspace{1cm} (2)

By choosing closed contours that are adequately far from the fracture tip in a homogenous material, it is possible to determine the J-integral with high accuracy. In contrast to isotropic materials, the J-integral in FGMs is typically path-dependent. There are two ways to fix this issue. The J-integral is modified in the first way as follows (Kawasaki et al. 1996):

$$J = \int_{A_i} (\sigma_{ij} u_{i,j} - W \delta_{1j}) q_j dA - \int_{A_i} W_{i,1} q dA$$  \hspace{1cm} (3)

Where:

$q$ is a function's weight and $A_i$ is the area bounded by the arbitrary closed contour. One accounts for the inhomogeneous effect of the material with graded qualities by adding the second component to the integral.

The second method is based on the simplified procedure (Guo et al. 2004), in which the mesh surrounding the crack tip is extremely small and the second term of the modified J-integral can be neglected. Additionally, in the case of FGMs, it is considered that the material characteristics are constant inside the components at the crack tip. The relationship indicated below can be used to calculate the SIFs:

$$K_I = \sqrt{JE_{tip}/(1-v_{tip}^2)}$$  \hspace{1cm} (4)

$E_{tip}$ and $v_{tip}$ represent for the FGM's Poisson's ratio and its elastic modulus, respectively, at the crack tip point.

3 FE MODELING IN FGM PLATES

FGM plate has been modeled using the ABAQUS finite element program (Dassault 2014). The USDFLD subroutine of the software FORTRAN allows for the definition of the fluctuation of the FGM physical parameters. The fluctuations of
the functionally graded material’s properties are taken into account based on an exponential law function model in the cracked plate. The subroutine has been programmed so that the domains are established with the appropriate characteristics of the material, including modulus E, ratio υ, conductivity k, thermal expansion α, specific heat Cᵥ, and density ρ.

The material properties for the FGM plate have been assumed to follow exponential functions given by:

\[ \rho(x) = \rho_1 e^{\gamma x} \text{ where } \gamma = \ln \left( \frac{\rho_2}{\rho_1} \right) \] (5a)

\[ E(x) = E_1 e^{\beta x} \text{ where } \beta = \ln \left( \frac{E_2}{E_1} \right) \] (5b)

\[ \alpha(x) = \alpha_1 e^{\eta x} \text{ where } \eta = \ln \left( \frac{\alpha_2}{\alpha_1} \right) \] (5c)

\[ k(x) = k_1 e^{\zeta x} \text{ where } \zeta = \ln \left( \frac{k_2}{k_1} \right) \] (5d)

\[ Cᵥ(x) = Cᵥ₁ e^{\lambda x} \text{ where } \lambda = \ln \left( \frac{Cᵥ₂}{Cᵥ₁} \right) \] (5e)

The properties of the various constituents of the FGM plate (metallic, ceramic) used are shown in Table 1.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Ceramic (Zirconia)</th>
<th>Metal (Titanium)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus E (GPa)</td>
<td>117</td>
<td>66.2</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.333</td>
<td>0.32</td>
</tr>
<tr>
<td>Density (kg/m²)</td>
<td>5600</td>
<td>4420</td>
</tr>
<tr>
<td>Coefficient of thermal expansion 10^-5 (1/K)</td>
<td>7.11</td>
<td>10.3</td>
</tr>
<tr>
<td>Thermal conductivity k (W/mK)</td>
<td>20.36</td>
<td>18.1</td>
</tr>
<tr>
<td>Specific heat Cᵥ (J/(kgK))</td>
<td>615.6</td>
<td>808.3</td>
</tr>
</tbody>
</table>

Source: Kobayashi et al. 1995
4 2D FGM PLATE WITH EDGE CRACK UNDER EFFECT OF TEMPERATURE

Now, a 2-Dimensional FE analysis is used to evaluate a FGM plate with an edge fracture of initial length a (Fig. 3a). The plate is fastened on the lower side and uni-axially loaded at a stress $\sigma_0 = 100$ MPa on the upper side.

To study the effect of temperature on the stress intensity factors in 2D FGM plates, we have adopted the Neumann type boundary conditions, which allow us to impose a given temperature on the left line of the plate (side of the plate from where a crack occurs) and to consider a phase of heating. This temperature varies from 400°C to 800°C depending on the case studied, while all the other lines are maintained at an initial temperature of $T_i = 300$°K, which is the ambient temperature. The mesh has been refined at the crack tip (Fig. 1.b), including quadratic elements with 20 nodes (C3D20R).

5 RESULTS AND DISCUSSIONS

The stress states induced by the thermo-mechanical stress fields corresponding to the ambient temperature and heating phases are shown in Figure 2. The Von Mises stress distribution for a particular crack length ($a/W = 0.15$) and different values of the imposed temperature are represented in Figure 2. It can be observed that the state of stress at $T=300$°K is more uniform than that at both $T=400$°K and 800°K. Von Mises stresses at the crack tip exhibits significant non-
uniformity as a result of the redistribution of stresses caused by the temperature increase in the plate around the crack. Due to the fact that this demonstrates a dangerous character during heating from the perspective of resistance, it is necessary to look at the FGM plates' fracture behavior under thermal loading.

Figure 2. Von Mises stress distribution at a/W = 0.15 and different temperatures in the 2D plate of FGM.

Figures 3 depict the Von Mises stress distributions for various values of cracking length at T=700°K. As the crack length increases, the plastic zone near the crack tip expands. These deformations are brought on by a rise in temperature and result in residual stresses.
Figure 3. Von Mises stress distribution at $T=700^\circ$K, for different crack lengths in the 2D plate of FGM.

Source: Authors.

Figure 4 presents the evolution of J-integral according with temperature, for different values $a/W$. J-integral drops as temperature rises. J-integral values are higher for the longest crack length ($a/W=0.40$) and lower for the shortest crack length ($a/W=0.10$). With longer cracks, the J-integral grows.

Figure 4. J-integral variation with respect temperature.

Source: Authors.

The evolution of J-integral according the dimensionless crack length $a/W$,
for different values of temperature is illustrate in figure 5. We can notice that the crack length increase leads to an increase in the integral J. Higher values of J-integral have been obtained for low temperatures and lower values for high temperatures.

Figure 5. J-integral variation with respect dimensionless crack length.

The variations of the SIFs KI and KII against temperature for various values of the dimensionless crack length a/W are shown in Figures 6 and 7, respectively. It is observable that when temperature rises, Mode I stress intensity components diminish. The opposite occurs for Mode II stress intensity factors. The subsequent behavior of the material may be noticeably impacted by the residual thermal stresses which strongly affect the properties of a ceramic-metallic FGM subjected at high temperatures. The difference in thermal expansion coefficients of the materials (ceramic/metallic) creates such residual stresses. The volume fractions of these material constituents vary exponentially, which accounts for the significant residual stresses observed in metal-rich materials.
The evolution of the SIFs $K_I$ and $K_{II}$ against the dimensionless crack length $a/W$, for different temperature values, are represented in Figure 8 and 9, respectively. Note that the SIF $K_I$ increases with the dimensionless crack length, while the stress intensity Factor $K_{II}$ decreases. Higher values of SIF $K_I$ have been obtained at low temperatures and lower values for high temperatures.
6 CONCLUSIONS

This paper presents a FEM-based study of the thermo-mechanical effects on the stress intensity factor in FGM. The variations of the functionally graded material properties follow exponential laws in the cracked plate, and the stress intensity factors have been evaluated using the new ABAQUS subroutine USDFLD. The following conclusions can be drawn:
The fractional gradient materials can exhibit gradual changes in density, thermal conductivity, thermal expansion, strength and modulus of elasticity through thickness, as a function of temperature. These changes will affect the area of plasticity near the crack, thus modifying the stress intensity factor.

Most materials expand when heated and contract when cooled. In a graded composite material, the different layers can have different coefficients of thermal expansion. This can lead to internal stresses (or residual stress) when there are temperature variations, which can influence the stability and overall deformation of the material, that is to say the difference in thermal expansion coefficients of the materials (ceramic/metallic) creates residual stresses.

The residual stresses in the plate around the crack are redistributed as a result of the rise in temperature, improving fracture resistance and lowering stress intensity factor values. Thermo-mechanical loads can have significant effects on the stress intensity factor in a 2D cracked plate of functional gradient material. Understanding and analyzing these effects is important for assessing crack propagation and durability of structures made from such functional gradient materials.
REFERENCES


