Double-diffusive convection of non-newtonian power-law fluids in an inclined porous layer

Convecção dupla difusiva de fluidos power-law não newtoniano sem uma camada porosa inclinada

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Saleh Khir  
PhD Student in Mechanical Engineering  
Institution: Department of Mechanical Engineering, Laboratory of Mechanics, Physics and Mathematical Modelling (LMP2M), University of Medea  
Address: Medea, 26.000, Algeria  
E-mail: iamkhirsaleh@gmail.com

Redha Rebhi  
Doctor in Mechanical Engineering  
Institution: Department of Mechanical Engineering, LERM-Renewable Energy and Materials Laboratory, Medea University  
Address: Medea, 26.000, Algeria  
E-mail: rebhi.redha@gmail.com

Mohamed Kezrane  
Doctor in Mechanical Engineering  
Institution: Department of Mechanical Engineering, Laboratory of Mechanics, Physics and Mathematical Modelling (LMP2M), University of Medea  
Address: Medea, 26.000, Algeria  
E-mail: mohamedkezrane@yahoo.fr

Faouzi Didi  
Doctor in Renewables Energies  
Institution: Laboratory of Renewable Energy and Materials (LREM), Department of common core, Faculty of Technology University Yahia Fares of Medea  
Address: Medea, 26.000, Algeria  
E-mail: didifouzi19@gmail.com

Selma Lounis  
Doctor in Process Engineering  
Institution: Department of Proceeding Engineering, Faculty of Technology, University of Blida 1  
Address: Blida, 09000, Algeria  
E-mail: lounisselma82@gmail.com
ABSTRACT
This paper presents a numerical study of Double-Diffusive convection within an inclined porous medium saturated by a non-Newtonian fluid. The power-law model is utilized for modelling the behavior of the flow in the porous layer. The given statement implies that the long side of the cavity experience thermal and solutal flux rates, whereas the other walls are impermeable and thermally isolated. The issue is characterized by a set of tightly linked non-linear differential equations, termed governing equations, encompassing the mass conservation equation (known as the continuity equation), the momentum equation, the energy equation, and the species equation. The relevant factors that govern the problem being investigated are the Rayleigh number, \( R_T \), the power-law index, \( n \), the angle of inclination, \( \Phi \), the cavity aspect ratio, \( A \), the Lewis number, \( Le \), the normalized porosity, \( \xi \), and the buoyancy ratio, \( N \), two types of cavity configuration have been studied: inclined cavity (i.e. \( \Phi \neq 0^\circ \)), then we have studied the case of a vertical cavity (i.e. \( \Phi = 90^\circ \)) where the buoyancy forces induced by the thermal and solutal effects are opposing each other and of equal intensity (\( N = -1 \)). A semi-analytical solution, valid for an infinite layer (\( A > 1 \)), is derived on the basis of the parallel flow approximation. A numerical approach utilizing the finite differences method was utilized to resolve the governing equations within the porous medium. It is demonstrated that both the inclination of the layer, \( \Phi \), and the power-law index, \( n \), have a strong influence on the strength of the intensity of flow, \( \Psi_0 \), the heat transfer rate, \( Nu \), and the mass transfer rate, \( Sh \), within the enclosure. A good agreement is found between the predictions of the parallel flow approximation and the numerical results obtained by solving the full governing equations.

Keywords: double-diffusive convection, power-law fluids, inclined porous layer, parallel flow.

RESUMO
Este artigo apresenta uma estratégia de controle aprimorada para inversores trifásicos de fonte Z multinível em cascata, com foco na implementação de técnicas de modulação por largura de pulso com eliminação seletiva de harmônicos (SHEPWM) para melhorar a eficiência. Os inversores de fonte Z estão ganhando destaque em diversas aplicações devido às suas vantagens inerentes, como maior capacidade de aumento de tensão e proteção inerente contra disparo. No entanto, alcançar o controle ideal em configurações multiníveis em cascata apresenta desafios que este estudo procura abordar. A estratégia de controle proposta otimiza os padrões de comutação do inversor, melhorando assim o seu desempenho geral. Ele permite o controle preciso da tensão de saída e reduz a distorção harmônica, garantindo que o inversor opere de maneira eficiente em diversas condições de carga. A metodologia de pesquisa envolve uma análise abrangente da técnica proposta, incluindo sua modelagem matemática e simulação utilizando ferramentas de software avançadas. Métricas de desempenho como distorção harmônica total, eficiência e resposta transitoria são avaliadas para quantificar as melhorias alcançadas. Os resultados obtidos demonstram a eficiência da estratégia de controle proposta. Este método reduz significativamente a distorção harmônica na tensão de saída, levando a uma melhor qualidade de energia. Além disso, a eficiência do inversor é melhorada, tornando-o adequado para aplicações que exigem conversão de energia de alto desempenho.
1 INTRODUCTION

Natural convection in an inclined fluid-saturated porous medium has been the subject of much research over the past decade, and studies in the literature include analytical and numerical studies, and experimental studies for different types of external and internal flows. Because of their widespread applications, such as in solar energy systems in energy sensor systems, arise in the design of pebble bed nuclear reactors, solar collectors, geothermal energy conversion, and the use of fibrous materials in thermal insulation of buildings and geophysical flows. Natural convection is a fundamental phenomenon in many types of collectors and receivers. The special feature of this study is the focus on flows driven by conditions of uniform heat and mass flow imposed along the perpendicular sidewalls of the porous layer. It is well known that natural convection phenomena are very sensitive to boundary conditions. From where we can find a very large number of studies in the literature related to this phenomenon. Comprehensive reviews of relevant studies on the topic in the bibliography [1-4]

Prasad and Kulacki [5] reported numerical results for the case where a constant heat flux is applied to one vertical wall, and the other vertical wall is isothermally cooled. Mamou [6] analyzed the thermal stability analysis of convection in a vertical porous subjected to different limits of heat and melt. The linear governing equations were solved numerically using the finite element method. It was reported that increasing the porosity of the porous medium delays the onset of oscillatory flows. Rebhi et al. [7] investigated the onset of nonlinear convection and Hopf bifurcation in a vertical porous filled with a binary fluid subject to constant horizontal temperature and concentration gradients. The same problem was reconsidered by Karimi-Fard et al. [8] for an inclined rectangular cavity. Mihir et al. [9] studied numerically and analytically the multiplicity of stable states in natural convection in rectangular porous layers with conduction isotherms. The results found that at subcritical Rayleigh numbers there is only one stable state. However, multiple stable states exist, some of which are unstable. Chan et al. [10] studied normal formation in porous media confined by surfaces
with rectangular boundaries at different temperatures. Osfer et al. [11] considered numerically and analytically the coupled natural convection in a vertical porous layer subjected to uniform heat and material flows from the side. Oremi et al. [12] studied the natural convection of a binary mixture confined in a long slightly inclined cavity heated from below. Both diffuse binary convection and induced convection were studied both theoretically and numerically. These authors proved the existence of multiple steady states for sufficiently small slopes about the vertical plane. Mamou and Vasseur [13] studied the onset of coupled diffusive heat and mass flows in an inclined fluid layer, when constant heat and mass fluxes are applied at the corresponding boundaries of the layer. They proved, on the basis of the parallel flow approximation, the existence of a subcritical Rayleigh number for the onset of convection of finite amplitude.

Inclined rectangles have received less attention, and one of the main contributions was by Vlasuk [14] performed a numerical integration of the governing equations and determined the heat transfer as a function of the angle of inclination. One of his important results was that the slope angle for maximum heat transfer for the ratio of unit dimensions and Darcy-Riley numbers in the range of 100-350 is approximately 50 degrees. Heat transfer through the porous layer is dominated by convection in the boundary layer system, where the Rayleigh number is very large. In the upper limit of the Rayleigh number, the Darcy flow model is inaccurate because the vertical velocity scale increases as $\frac{Ra^2}{3}$. Gravity-induced thermal diffusion in a rectangular vertical cavity subjected to horizontal thermal gradients as studied by Marcoux et al. [15] for the case of opposing and equal thermal buoyancy forces. The supercritical Rayleigh number for the onset of convection from rest is predicted on the basis of linear stability theory, in terms of the aspect ratio of the cavity. In explaining the main features of the heat and mass transfer phenomenon discussed by Bejan [16], where pure measurement arguments were used to identify and sort out the most fundamental metrics that characterize the flow, temperature and concentration fields in the immediate vicinity of a single submerged vertical surface in a porous medium with different temperature and concentration. A mathematical solution to the equations of mass, energy and appropriate momentum equations led Holst and Aziz [17]. These authors concluded that the maximum convection should occur at an angle
of 40° and Rayleigh numbers of 50-60, provided the aspect ratio is one. Alloui et al. [18] numerical study of natural convection in an inclined porous medium. Their results indicated that the effect of the drag coefficient on the existence of multiple steady-state solutions possible for a surface slightly tilted about the horizontal. Vasseur et al. [19] determined the natural convection in a two-dimensional (2D) oblique porous layer with uniform heat flow from two opposite walls while the other walls were studied analytically and numerically. The authors showed that good agreement was found between analytical predictions and numerical simulations. The multiple steady-state solutions for slightly inclined porous materials, as investigated analytically by Walsh and Dulieu [20]. In a more recent numerical study of natural convection in a fluid-saturated rectangular porous material analyzed by Moya et al. [21]. Their studies revealed that when the bottom wall is at a higher temperature and in a horizontal or near-horizontal position.

For the Newtonian case, flows in such configurations have been the subject of many experimental and numerical studies. Ozoe and Sayama [22] experimentally and numerically on heat transfer in inclined cavities for a range of Rayleigh numbers and tilt angles. Ozu and Sayama [23, 24] in an experimental and numerically calculated values of the Nusselt number for natural convective heat in square and rectangular channels.

Recently, Kim et al. [25] studied transient buoyant convection in a square cavity subjected to hot and cold temperatures on the vertical sidewalls of thin Ostwald-DeWeyl type shear force law fluids. The study concludes that for high Rayleigh $Ra = 10^5 - 10^7$ and Prandtl numbers $Pr = 10^2 - 10^4$, convective activity intensifies with decreasing power-law index $n$ resulting in enhanced overall heat transfer coefficients. Bian et al. [26] performed numerical studies on the rectangular cavity of natural convection filled with a porous medium saturated with a non-Newtonian fluid, and their results found that the power-law index $n$ seems to have no effect on the extreme angle, for example Nusselt number peaks occur at about 135° for various power-law indices when $R = 100$. Numerical and analytical study of natural convection in a vertical porous cavity filled with a non-Newtonian binary fluid, characterized by a power-law model, by Ben Khalifa et al. [27]. The heat and mass transfer rates are in good agreement between the analytical predictions and numerical simulations obtained. Lamsadi et al. [28] numerically
simulated the natural convection of a non-Newtonian fluid in a vertical cavity. Their results showed that shear-thinning fluid contributes to fluid flow, shear-thinning fluid decreases fluid flow, and shear-thinning fluid increases heat transfer. The natural convection of non-Newtonian fluid in a tilted two-dimensional rectangular cavity investigated by Khezzar et al. [29]. The results showed that increasing the tilt angle from horizontal leads to a sudden decrease in heat transfer rate. Experimental and numerical studies have also confirmed the increase in natural convective strength in rectangular bundles of shear-thinning fluids on microemulsion pastes by Inaba et al. [30] in a Rayleigh-Bénard configuration.

Previous work on natural convection of binary fluids is concerned with the onset of motion in a horizontal porous layer subject to vertical temperature and concentration gradients. In practice, many researchers have presented analytical and experimental results for a rectangular cavity with vertical walls at constant temperatures, where the walls are insulated horizontal. [31-37].

This study primarily concentrated on examining the Double-diffusive convection phenomena occurring within an inclined porous layer containing a non-Newtonian fluid and subjected to bottom heating and salting. Particularly focusing on the behavior of non-Newtonian fluids using the power-law model. This study is an extension of the prior work conducted by Khir et al. [38], aiming to explore the impact of non-zero inclination angles in the system, previous research in this domain has predominantly focused on the scenario of an inclined enclosure being uniformly heated [9,26], and thus did not account for the impact of both thermal and solutal flux rates experienced within the enclosure. The resolution of the comprehensive nonlinear governing equations was achieved through the application of a time-accurate finite difference method. Furthermore, a semi-analytical solution for shallow enclosures was derived, employing the parallel flow approximation. The outcomes are expressed in relation to flow intensity, $\Psi_0$, heat transfer rate, $Nu$, and mass transfer rate, $Sh$, and their dependence on the governing parameters is thoroughly examined. The numerical findings presented in this context contribute to understanding the influence of various parameters, such as the power-law index, $n$, the Lewis number, $Le$, the buoyancy ratio, $N$, and the angle of inclination, $\Phi$, on natural convection within an inclined and vertical enclosure.
2 PROBLEM DEFINITION AND MATHEMATICAL FORMULATION

Figure 1 depicts the two-dimensional geometric configuration of the flow. We analyze a scenario in which a rectangular inclined cavity, with dimensions specified by height $H'$ and length $L'$, contains a porous medium saturated with a non-Newtonian power-law fluid. The coordinate axes $x'$ and $y'$ are aligned at an angle $\Phi$ with the horizontal and vertical orientations, respectively. The long side walls of the layer are subjected to a constant heat flux $q'$ and solute flux $j'$, whereas the other walls are assumed to be adiabatic and impermeable.

In alignment with earlier findings clarified by Pascal [39, 40], the model describing laminar flow of a non-Newtonian power-law fluid through a porous medium is expressed as follows:

$$
\mathbf{v}' = -\frac{K}{\mu'_a} \nabla p'
$$

Where

$$
\mu'_a = \varepsilon (u'^2 + v'^2)^{(n-1)/2}
$$

\[1\]

\[2\]
\[ \varepsilon = \frac{2h}{8^{(n+1)/2} \left( K \phi \right)^{(n+1)/2} \left( n/(1+3n) \right)^n} \quad (3) \]

In the equations above, \( V' \) is the superficial velocity, \( \phi \) and \( K \) the porosity and the permeability of the porous medium respectively, \( \mu'_{\text{a}} \), the apparent viscosity, \( \varepsilon \) is a parameter in power-law model, \( h \) the consistency index and \( n \) the power-law index. The rheological parameters \( h \) and \( n \) are assumed to be temperature independent.

The equations governing the heat flux \( q' \) and mass flux \( j' \) concerning the thermal and solute gradients within the binary fluid mixture are clarified by De Groot and Mazur [41] as follows:

\[ \bar{q}' = -k \nabla T' \quad (4) \]
\[ \bar{j}' = -\rho D \nabla S' \quad (5) \]

where, \( \rho \) is the density of the non-Newtonian fluid. \( T' \) and \( S' \) represent the temperature and concentration of the fluid at a specific point within the system, while \( k \) and \( D \) signify the thermal conductivity and molecular diffusion coefficient of the species.

The non-Newtonian fluids considered here are presumed to adhere to the Boussinesq approximation which has been used by Amari et al. [7], Bian et al. [26], wherein the density variation with temperature and concentration is described through the state equation, as follow

\[ \rho = \rho_0 \left[ 1 - \beta_T' (T' - T'_0) - \beta_S' (S' - S'_0) \right] \quad (6) \]

where \( T'_0, S'_0 \) and \( \rho_0 \) represent respectively, the temperature, concentration, and mass density of reference. \( \beta_T', \beta_S' \) respectively are the thermal and solutal expansion coefficients.

They are defined by

\[ \beta_T' = -\frac{1}{\rho_0} \left( \frac{\partial \rho}{\partial T} \right)_{P,S}, \quad \beta_S' = -\frac{1}{\rho_0} \left( \frac{\partial \rho}{\partial S} \right)_{P,T} \quad (6) \]
The equations describing the conservation of mass, momentum, energy, and concentration for the current problem are, in respective order

\[ \nabla \vec{V}' = 0 \]  

\[ \vec{V}' = -\frac{K}{\mu_a} [\nabla p' + \rho \vec{g}] \]  

\[ (\rho C)_f \frac{\partial T'}{\partial t'} + (\rho C)_p J(\Psi', T') = k \nabla^2 T' \]  

\[ \phi \frac{\partial S'}{\partial t'} + J(\Psi', S') = D \nabla^2 S' \]  

Herein, \( g \) is the gravitational acceleration, \((\rho C)_f\) and \((\rho C)_p\) are the heat capacities of fluid and saturated porous medium.

Equations (8)-(11) are put in a dimensionless form by defining a new set of variables

\[ (x, y) = \left( \frac{x', y'}{H'} \right) \quad (u, v) = \left( \frac{u', v'}{H' / \alpha} \right) \quad t = t' \alpha \left( \sigma H'^2 \right) \]

\[ \psi = \psi' / \alpha \quad T = \left( T' - T_o' \right) / \Delta T' \quad \Delta T' = q' H' / k \]

\[ S = \left( S' - S_o' \right) / \Delta S' \quad \Delta S' = j' H' / \rho D \quad \mu_a = \mu'_a / \varepsilon \left( \alpha / H' \right)^{a-1} \]

where \( \alpha = k / (\rho C)_f \) is the thermal diffusion coefficient, and \( \sigma = (q C)_p / (q C)_f \) is the heat capacity ratio.

By using the above equations, the Boussinesq approximation, and eliminating the pressure term in Eq. (9) The dimensionless equations governing the convective flow are provided as follows:

\[ \nabla^2 \psi = -\mu_a^{-1} \left[ \frac{\partial^2 \psi}{\partial y^2} \frac{\partial u_a}{\partial y} + \frac{\partial^2 \psi}{\partial x^2} \frac{\partial u_a}{\partial x} + R_T \left( \frac{\partial T}{\partial x} + N \frac{\partial S}{\partial x} \right) \cos \Phi - \left( \frac{\partial T}{\partial y} + N \frac{\partial S}{\partial y} \right) \sin \Phi \right] \]  

\[ \nabla^2 T = \frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \]
\[
Le^{-2} \nabla^2 S = \xi \frac{\partial S}{\partial t} + \frac{\partial \Psi}{\partial y} \frac{\partial S}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial S}{\partial y}
\]

(12)

\[
\mu_a = \left[ \left( \frac{\partial \Psi}{\partial x} \right)^2 + \left( \frac{\partial \Psi}{\partial y} \right)^2 \right]^{n-1}
\]

(13)

The dimensionless parameters who were extracted from Eqs. (13)-(15) are Rayleigh number \( R_T = K \rho_0 g \beta' T' (H'/\alpha)^n / \varepsilon \), the Lewis number \( Le = \alpha / D \), the buoyancy ratio \( N = \beta' \delta S' / \beta' T' \Delta T' \), the normalized porosity \( \xi = \phi / \sigma \), and \( \Psi \) is a dimensionless stream function defined as

\[
u = \frac{\partial \Psi}{\partial y}, \quad \nu = -\frac{\partial \Psi}{\partial x}
\]

(14)

The boundary conditions, expressed in dimensionless terms, are defined as follows:

\[
\Psi = \frac{\partial \Psi}{\partial x} = 0, \quad \frac{\partial T}{\partial x} = \frac{\partial S}{\partial x} = 0 \quad \text{and} \quad x = \pm \frac{A}{2}
\]

(15)

\[
\Psi = \frac{\partial \Psi}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = \frac{\partial S}{\partial y} = -1 \quad \text{and} \quad y = \pm \frac{1}{2}
\]

where \( A = L'/H' \) is the cavity aspect ratio.

In the current notation, at any given position \( x \) along the long walls, both local and average Nusselt and Sherwood numbers, denoted as \( Nu_x \), \( Nu_m \), \( Sh_x \), and \( Sh_m \), respectively, are expressed as:

\[
\begin{align*}
Nu_x^{-1} &= T_{(x,-1/2)} - T_{(x,1/2)}; \quad Sh_x^{-1} = S_{(x,-1/2)} - S_{(x,1/2)} \\
Nu_m &= A^{-1} \int_{-A/2}^{+A/2} Nu_x \, dx; \quad Sh_m &= A^{-1} \int_{-A/2}^{+A/2} Sh_x \, dx
\end{align*}
\]

(16)
3 NUMERICAL SOLUTION

The finite difference method is employed to numerically solve the governing equations (13)-(15). The energy and concentration equations are discretized utilizing a second-order centred scheme. At each time increment, the alternating directions implicit method (ADI) of Peaceman and Rachford [42], yields two tri-diagonal matrix systems for resolution, one arising from the implicit discretization in the $x$-direction and the other from the implicit discretization in the $y$-direction. To address the discretized motion equation at every time increment, we employ the successive over-relaxation method (SOR), which is an explicit technique providing the value of $\Psi$ at the instance $(n + 1)\Delta t$ directly. The convergence criterion for solving Eq. (13) is outlined.

$$\frac{\sum \sum |\Psi^{i+1}_{i,j} - \Psi^i_{i,j}|}{\sum \sum |\Psi^i_{i,j}|} \leq 10^{-6}$$

(20)

The findings demonstrate an excellent agreement between the semi-analytical and numerical solution. Numerical assessments were conducted with diverse mesh sizes under identical conditions to establish the optimal balance between result accuracy and computational efficiency, as presented in Table 1.

Figures 2 and 3 presents the contours of streamlines, $\Psi$, Temperature, $T$, Concentration, $S$, and apparent viscosity, $\mu_a$, obtained numerically, for a fluid with power-law index, $n = 1.2$, $R_T = 50$, $Le = 5$, $N = -0.1$, and different angle of inclination $\Phi = 0^\circ$, $60^\circ$, $90^\circ$, and $120^\circ$ presented in Figs. 2(a)-(c), and for a special case for a vertical cavity heated from the side in Figs. 3(a)-(c) where $R_T = 100$, $Le = 5$, $N = -1$, and $\Phi = 90^\circ$, and different values of power-law index $n$. The intensity of flow, $\Psi_0$, in the core region of the enclosure is essentially parallel while the temperature $T$, and the concentration $S$ in the core of the cavity are linearly stratified in the $x$-direction. The apparent viscosity, $\mu_a$, lines are uniformly distributed between the center, and the edges of the cavity and the extremum value, and it’s clearly constant when $n = 1$. 
The numerical findings depicted in Figures 2 and 3 validate the theoretical assumption of parallel flow, indicating that the flow patterns within the central region of the cavity remain parallel regardless of the power-law index, $n$, when the aspect ratio $A >> 6$. For this reason, most of the numerical results reported here were obtained for $10 \geq A \geq 6$ with typically $250 \times 150$ mesh points.
Table 1. Convergence tests for specified parameters, \( A = 6, \ Le = 5, \ N = -0.1, \ R_T = 50, \) and \( n = 1.2, \) and different angles of inclination \( \Phi \).

<table>
<thead>
<tr>
<th>Grids</th>
<th>( \Psi_0 )</th>
<th>( Nu )</th>
<th>( Sh )</th>
<th>( \Psi_0 )</th>
<th>( Nu )</th>
<th>( Sh )</th>
<th>( \Psi_0 )</th>
<th>( Nu )</th>
<th>( Sh )</th>
<th>( \Psi_0 )</th>
<th>( Nu )</th>
<th>( Sh )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi = 0^\circ )</td>
<td>1.971</td>
<td>2.160</td>
<td>4.997</td>
<td>2.074</td>
<td>2.346</td>
<td>5.447</td>
<td>1.741</td>
<td>2.045</td>
<td>5.334</td>
<td>1.261</td>
<td>1.598</td>
<td>4.765</td>
</tr>
<tr>
<td>( \Phi = 60^\circ )</td>
<td>1.925</td>
<td>2.173</td>
<td>5.098</td>
<td>2.054</td>
<td>2.372</td>
<td>5.627</td>
<td>1.717</td>
<td>2.060</td>
<td>5.529</td>
<td>1.246</td>
<td>1.601</td>
<td>4.937</td>
</tr>
<tr>
<td>( \Phi = 90^\circ )</td>
<td>1.942</td>
<td>2.173</td>
<td>5.142</td>
<td>2.042</td>
<td>2.381</td>
<td>5.716</td>
<td>1.704</td>
<td>2.065</td>
<td>5.626</td>
<td>1.227</td>
<td>1.601</td>
<td>5.021</td>
</tr>
<tr>
<td>( \Phi = 120^\circ )</td>
<td>1.937</td>
<td>2.172</td>
<td>5.158</td>
<td>2.037</td>
<td>2.384</td>
<td>5.754</td>
<td>1.698</td>
<td>2.066</td>
<td>5.669</td>
<td>1.221</td>
<td>1.601</td>
<td>5.058</td>
</tr>
</tbody>
</table>

Source: Authors

To verify the accuracy of the numerical solutions presented herein, Tables 2 and 3 display the acquired numerical results, comprising the center stream function value \( \Psi_0 \), alongside the local Nusselt \( Nu \) and Sherwood \( Sh \) numbers. A thorough examination against the results documented by Bian et al. [26] and Ben Khelifa et al. [27] demonstrates a high degree of concordance.

Table 2. Comparison of \( \Psi_0 \), and \( Nu \) with previous numerical studied of Bian et al. [26] for \( R_T = 100, \ n = 1.2, \ N = 0, \) and \( A = 4 \) with various inclination angle \( \Phi \).

<table>
<thead>
<tr>
<th>( \Phi )</th>
<th>( \Psi_0 )</th>
<th>( Nu )</th>
<th>( Vs ) Bian et al. [26]</th>
<th>( Vs ) Bian et al. [26]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>2.966</td>
<td>3.058</td>
<td>0.47 %</td>
<td>0.47 %</td>
</tr>
<tr>
<td>60°</td>
<td>2.693</td>
<td>3.195</td>
<td>0.30 %</td>
<td>0.96 %</td>
</tr>
<tr>
<td>120°</td>
<td>1.400</td>
<td>1.860</td>
<td>0.36 %</td>
<td>0.50 %</td>
</tr>
</tbody>
</table>

Source: Authors

Table 3. Comparison of \( \Psi_0 \), \( Nu \) and \( Sh \) with previous numerical studied of Ben Khelifa et al. [27] for \( R_T = 200, \ N = 1, \ Le = 10, \ A = 4, \) and \( \Phi = 90^\circ \) with various values of \( n \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \Psi_0 )</th>
<th>( Nu )</th>
<th>( Sh )</th>
<th>( Vs ) Ben Khelifa et al. [27]</th>
<th>( Vs ) Ben Khelifa et al. [27]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>3.968</td>
<td>11.396</td>
<td>43.360</td>
<td>1.34 %</td>
<td>0.44</td>
</tr>
<tr>
<td>1.0</td>
<td>2.413</td>
<td>4.176</td>
<td>13.837</td>
<td>5.93</td>
<td>7.38</td>
</tr>
<tr>
<td>1.6</td>
<td>1.876</td>
<td>2.538</td>
<td>7.958</td>
<td>7.91</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Source: Authors
4 ANALYTICAL SOLUTION

This section delves into a semi-analytical resolution for the current problem. Particularly, in scenarios of a porous layer with a significant aspect ratio \( A \gg 1 \), as previously discussed by researchers such as Vasseur et al. [43] and Mamou et al. [44], it is reasonable to assume a mainly parallel flow pattern in the \( x \)-direction within the central enclosure region. Thus, only the velocity component \( U(y) \) in that direction exists.

As a result, the stream function \( \Psi \) becomes a function of the \( y \) ordinate only. We can then write:

\[
\Psi(x, y) = \Psi(y) \tag{17}
\]

the temperature and concentration profiles are given by:

\[
\begin{align*}
T(x, y) & = C_T x + \theta_T (y) \\
S(x, y) & = C_S x + \theta_S (y)
\end{align*} \tag{18}
\]

where \( C_T \) and \( C_S \) are unknown constants temperature and concentration gradients in the \( x \)-direction.

Substituting Equations (21) and (22) in the governing equations (13)-(15), we get the following ordinary differential equations:

\[
\frac{d}{dy} \left[ \frac{d\Psi}{dy} \frac{d\Psi}{dy} \right] = -R_T \left[ \frac{dT}{dx} + N \frac{dS}{dx} \cos \Phi - \left( \frac{dT}{dy} + N \frac{dS}{dy} \right) \sin \Phi \right] \tag{19}
\]

\[
\frac{d^2 \theta_T}{dy^2} = C_T \frac{d\Psi}{dy} \tag{20}
\]

\[
\frac{d^2 \theta_S}{dy^2} = C_S Le \frac{d\Psi}{dy} \tag{21}
\]

By using the boundary conditions, Eq. (18), and performing a first integration into Eqs. (24), and (25), it is found that:
\[ \frac{d\theta_c}{dy} = C_T \Psi - 1 \]  

(22)

\[ \frac{d\theta_s}{dy} = LeC_S \Psi - 1 \]  

(23)

Through Eqs. (23), (26) and (27) it is found that the momentum equation can be formulated as follows:

\[
\frac{d}{dy} \left[ \frac{d\Psi}{dy} \right]^{n-1} - R_T \left( C_T + NLeC_S \right) \Psi \sin \Phi = -R_T \left[ \left( C_T + NC_S \right) \cos \Phi + (N + 1) \sin \Phi \right]
\]  

(24)

It is evident that the boundary conditions in the x-direction, as specified by Eq. (18a), cannot be directly applied under the parallel flow approximation. Nevertheless, as elucidated previously by Trevisan and Bejan [11], we can enforce equivalent conditions for energy and mass fraction fluxes. Consequently, the thermal and species transport across a transversal section at any cross-section \( x \) must equate to zero, leading to the establishment of the following relationships:

\[
\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\partial T}{\partial x} \, dy + \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\partial \Psi}{\partial y} T \, dy = 0
\]

\[
\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\partial S}{\partial x} \, dy + Le \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\partial \Psi}{\partial y} S \, dy = 0
\]  

(25)

Eq. (29) can be combined with Eqs. (26), (27) and (22) to obtain the following expressions of \( C_T \) and \( C_S \):

\[
C_T = \frac{\int_{-\frac{1}{2}}^{\frac{1}{2}} \Psi(y) \, dy}{1 + \int_{-\frac{1}{2}}^{\frac{1}{2}} \Psi^2(y) \, dy}, \quad C_S = \frac{Le \int_{-\frac{1}{2}}^{\frac{1}{2}} \Psi(y) \, dy}{1 + Le^2 \int_{-\frac{1}{2}}^{\frac{1}{2}} \Psi^2(y) \, dy}
\]  

(26)

The equation (28), with initial values estimated for \( C_T \) and \( C_S \) and subject to boundary conditions outlined in Eq. (18), is numerically solved employing the TRIDAG method to determine the stream function field \( \Psi \). Initially, this equation is
discretized via a finite difference approach. Subsequently, utilizing the Simpson integration scheme, new values for $C_T$ and $C_S$ are derived by executing the numerical integration described in Eq. (30). This iterative process persists until convergence is attained for $C_T$, $C_S$, and $\Psi$. Intentionally chosen initial solutions are utilized, coupled with a relaxation technique, to expedite convergence and stabilize the sought-after solutions. Following this, the temperature and concentration fields, $T$ and $S$, are evaluated by solving Eqs. (26) and (27), along with their corresponding boundary conditions detailed in Eq. (18b). Consequently, the Nusselt and Sherwood numbers are computed using Eq. (19). Remarkably, this semi-analytical method significantly reduces computational time compared to the numerical procedure required for solving the complete governing equations (13)-(15).

Figures 4(a) and (b) depict the values of $C_T$ and $C_S$, representing the constant temperature and concentration gradient in the $x$-direction, respectively, for $R_T = 50$, $Le = 5$, and $N = -0.1$, plotted as functions of the inclination angle $\Phi$ and power-law index $n$. In the graph, solid lines corresponding to positive (negative) values of $\Phi$ indicate the natural (antinatural) solutions, while the dashed-dotted-dotted lines represent the unstable solutions, an occurrence similarly noted by Alloui et al. [18]. The graphs clearly demonstrate that, for a fixed inclination angle $\Phi$, the constant temperature and concentration gradients $C_T$ and $C_S$, respectively, diminish consistently with a decrease in $n$. At sufficiently large inclinations $\Phi$, within a given configuration of governing parameters. The graph indicates that the solutions for $C_T$ and $C_S$ are singular. For minor tilt angles $\Phi$, there are two stable solutions and one unstable solution. Within the stable solutions, one is a natural solution, and the other is an antinatural solution.
5 RESULTS AND DISCUSSION

The analysis of double-diffusive convection in the considered porous layer is characterized by the specification of several parameters, including the Rayleigh number $R_T$, the power-law index $n$, the inclination angle of cavity $\Phi$, the buoyancy ratio $N$, and the Lewis number $Le$. The semi-analytical solution provides predictions that are verified by numerical analyses of the full governing equations for a tall cavity ($A >> 1$). The study encompasses a range of parameters: 0.4 to 1.6 for $n$, which include shear-thinning ($n < 1$) and shear thickening ($n > 1$) fluids, -180° to 180° for $\Phi$, $10^{-2}$ to 10 for $Le$, and -5 to 5 for $N$.

Our research primarily focuses on investigating convection in an inclined cavity heated and salted from the bottom, which is discussed initially. Subsequently, we examine the case of a vertical cavity.

5.1 A- CASE OF AN INCLINED LAYER

Figures 5(a)-(d) depict profiles of horizontal velocity $U$, apparent viscosity $\mu_a$, temperature $T$, and concentration $S$ at the center of the layer $x = 0$ for specified values of $R_T = 50$, $Le = 5$, $N = -0.1$, $n = 1.2$, and varying inclination angles $\Phi$. The results are showcased within the range $-0.5 \leq y \leq 0.5$. The semi-analytical solution, depicted by solid lines, closely aligns with the numerical results, represented by solid circles, thereby indicating a favorable agreement between the current semi-analytical approach and the numerical solution. Figure 4 illustrates a
rise in velocity concurrent with an increase in the inclination angle $\Phi$ until it reaches the maximal angle $\Phi_{max}$ (refer to Figure. 6). Subsequently, commencing from this angle, the velocity begins to decline as the inclination angle $\Phi$ increases until it diminishes entirely at $\Phi = 180^\circ$. This phenomenon arises from the amplification of the inclination angle, resulting in the gradual dissipation of both temperature and mass gradients. A similar trend is observed concerning the apparent viscosity $\mu_a$, which decreases until the cessation of convective motion at $\Phi = 180^\circ$. Additionally, it’s noteworthy that the apparent viscosity $\mu_a$ is more pronounced at the cavity edges and absent in the central region due to the fluid exhibiting dilatant behavior with a power-law index of $n = 1.2$. In Figures 5(c) and (d), it is evident that the temperature and concentration profiles demonstrate a growth with the increasing inclination angle, $\Phi$. Additionally, at $\Phi = 180^\circ$ show a linear behavior of temperature and concentration due to the absence of temperature and mass gradient.
Figure 5. Effect of inclination angle, $\Phi$, on (a) the horizontal velocity, $U$, (b) apparent viscosity, $\mu_a$, (c) temperature, $T$, and (d) concentration, $S$.

The effects of the inclined angle, $\Phi$, and the power-law index, $n$, on the strength of convection $\Psi_0$, the heat transfer rate, $Nu$, and the solute transfer rate, $Sh$, is presented in Figures 6(a)-(c) for $R_T = 50$, $Le = 5$ and $N = -0.1$. Results are obtained for values of the power-law index, $n$, between 0.6 to 1.4. The curves depicted in the graphs are the predictions of the present asymptotic/numerical nonlinear models. The solid lines correspond to stable branches, while the dashed-dotted-dotted lines denote unstable branches. The numerical solutions derived from the complete governing equations are depicted using solid circles. Remarkably, a noteworthy concordance is evident between the outcomes generated by these two nonlinear theories. Initially, it is apparent that concerning the strength of convection $\Psi_0$ curve, either the upper or lower half independently resembles half of a heart shape. Furthermore, regarding the Nusselt $Nu$ and Sherwood $Sh$ numbers, combining the first and second quadrants yields a butterfly-like shape, a phenomenon similarly noted by Sen et al. [9] and Alloui et al. [18].
Figure 6. Effects of inclination angle $\Phi$ and power-law index $n$, for $R_T = 50$, on: (a) stream function at the center of the cavity $\Psi_0$, (b) Nusselt number $Nu$, and (c) Sherwood number $Sh$.

In general, it is seen from Figures 6(a)-(c) that for a given inclined angle, $\Phi$, both the strength of convection $\Psi_0$, the resulting heat transfer $Nu$, and solute transfer $Sh$ decrease as the power-law index, $n$, increases, this aligns with the observations in Figures 4(c) and 4(b), where the substantial circulation diminishes $C_T$ and $C_S$ while augmenting $Nu$ and $Sh$, as anticipated. The curve depicting the strength of convection, $\Psi_0$, indicates that the first and third (or second and fourth) quadrants delineate the natural (or antinatural) flows. Within the first (or third) quadrant, positive (or negative) $\Psi_0$ denotes counterclockwise (or clockwise) motion. The plot effectively demonstrates that as the inclination angle decreases from $180^\circ$ to $\Phi_{CR}$, as illustrated in the zoomed image in Figure 6(a), the strength of convection $\Psi_0$, the Nusselt number $Nu$, and the Sherwood number $Sh$ initially rise, reaching a peak at $\Phi = \Phi_{max}^{\Psi_0}$, $\Phi = \Phi_{max}^{Nu}$, and $\Phi = \Phi_{max}^{Sh}$, respectively, where $\Phi_{max}^{\Psi_0}$, $\Phi_{max}^{Nu}$, and $\Phi_{max}^{Sh}$ are indicated in Fig. 6(d), and subsequently decline. At sufficiently large inclinations $\Phi$, within a given
configuration of governing parameters, the graph show that the solutions for \( \Psi_0 \), \( Nu \), and \( Sh \) are singular. For slight tilt angles \( \Phi \), two stable solutions and an unstable one are possible. Among the stable solutions, one is natural solution, while the other is antinatural solution. Eventually, it is noted that the power-law index \( n \) additionally influences the range within which multiple solutions manifest. Hence, Figs. 6(a)-(c) and 4(a)-(b) illustrate that, for a specified value of \( n \), multiple steady states are viable within the range of \( -\Phi_{CR} < \Phi < \Phi_{CR} \).

Figure 7 illustrates the variation of the critical tilt angle \( \Phi_{CR} \) relative to the power-law index \( n \). It is observed that with an increase in the power-law index \( n \), the interval of tilt angles conducive to multiple steady states diminishes notably. Specifically, when \( n \) exceeds 1.8 and falls below 0.4, \( \Phi_{CR} \) assumes values of \( \Phi_{CR} = -11.8 \) and \( \Phi_{CR} = -35 \), respectively.

Figures 8(a)-(c) displays the variation of the strength convection \( \Psi_0 \), Nusselt number \( Nu \), and Sherwood number \( Sh \) as a function of the power-law index \( n \) for \( R_T = 50 \), \( Le = 5 \) and \( N = -0.1 \), with various inclination angles \( \Phi \). The findings reveal that at inclination angles conducive to high natural convection strength (\( \Phi = 0^\circ \) and \( 60^\circ \)), there is a notable decline in \( \Psi_0 \), \( Nu \), and \( Sh \) as the power-law index \( n \) increases. Conversely, at significantly large \( \Phi \) values (\( 160^\circ \) and \( 180^\circ \)), where the strength of convection, heat transfer, and solute transfer are predominantly conductive, \( \Psi_0 \), \( Nu \), and \( Sh \) remain constant regardless of the value of \( n \).
depicted in Figure 8, the elevation of the power-law index \( n \) diminishes the strength of convection \( \Psi_0 \), heat and solute transfer. Finally, at \( \Phi = 0^\circ \), as mentioned in a previous study [38], the value of \( \Psi_0 \) approaches 0.5, and Nu and Sh converge to 1.061 and 2.027, respectively, for a sufficiently large \( n \).

Figure 8. Effects of power-law index \( n \), and inclination angle \( \Phi \), for \( R_T = 50 \), on: stream function \( \Psi_0 \) at the center of the cavity, (b) Nusselt number \( Nu \), and (c) Sherwood number \( Sh \).

5.2 B- CASE OF A VERTICAL LAYER

In this section, we investigate a vertical porous layer (i.e., an inclined layer with \( \Phi = 90^\circ \)) where the two opposing sides are subjected to uniform heat and mass flux, while the other two sides are adiabatic and impermeable.

Figure 9 depict the influence of the power-law index \( n \) on vertical velocity \( U \), apparent viscosity \( \mu_a \), temperature \( T \), and concentration \( S \) profiles at the mid-height of the porous layer (i.e., \( x = 0 \)) for \( 0.6 < n < 1.4 \), with \( R_T = 100 \), \( N=-1 \), and \( Le=5 \).
These figures demonstrate that the asymptotic semi-analytical solution, represented by solid lines, aligns closely with the numerical results denoted by solid circles. In Figure 9(a), the vertical velocity visibly diminishes with increasing \( n \), rendering it negligible at the center and pronounced at the cavity's sides. This trend is mirrored in Figure 9(b) concerning the apparent viscosity, where it is large at the sides for shear-thickening fluids \((n > 1)\) and minimal for shear-thinning fluids, while in the center, the inverse holds true. Notably, in the case of Newtonian fluid \((n = 1)\), the apparent viscosity remains constant \((\mu_a = 1)\). Figures 9(c) and (d) reveal that both temperature and concentration profiles exhibit enhancement with rising values of the power-law index, \( n \). Furthermore, all curves exhibit a consistent slop at the boundary as a result of the constant heat and mass flux applied to the vertical walls.

Figure 10 illustrates the evolution of the strength of convection \( \Psi_0 \), and the Nusselt \( Nu \) number, and Sherwood \( Sh \) numbers as a function of the Rayleigh number \( R_T \), with various values of the power-law index \( n \) for \( Le = 5, N = -1 \). For
high $R_T$ values, only the asymptotic trend of the semi-analytical solution is accounted for due to the numerical results deviating from the semi-analytical solution at high Rayleigh numbers. Generally, it is observed from Figures 10(a) and (b) that for a given Rayleigh number $R_T$, both the strength of convection $\Psi_0$, and the Nusselt number $Nu$, and Sherwood $Sh$ number increase as the power-law index, $n$, decreases. It is also noted that $\Psi_0$ increases monotonically with $R_T$, as depicted in Figure 10(a), whereas in Figure 10(b), the Nusselt number $Nu$, and Sherwood number $Sh$ rise asymptotically towards constant values $Nu=Sh=6.129, 5.600, 5.278, 5.065, 4.913$ for $n=0.6, 0.8, 1.0, 1.2, 1.4$ respectively. These constants depend solely on the value of the power-law index $n$, irrespective of the values of $N$ and $Le$.

Figure 10. Large Rayleigh number $R_T$ and power-law index $n$ effects on (a) the flow intensity $\Psi_0$ and (b) the Nusselt $Nu$ and Sherwood $Sh$ number.

![Graphs showing the effects of $R_T$ and $n$ on $\Psi_0$, $Nu$, and $Sh$](source: Authors)

The variations of $\Psi_0$, $Nu$, and $Sh$ as a function of Lewis number $Le$, and the power-law index $n$, for $R_T = 100$ and $N = -1$, are exhibited in Figures 11(a)-(c). The subcritical values of $Le$ for the selected $R_T$ and $N$ are outlined in Table 5. Between these limits, heat and mass transfer occur solely via conduction. Excluding these boundaries, two regions emerge in each figure where two convective solutions originate from the fluid's rest state: one is stable (solid line), and the other is unstable (dashed line). From Figures 11(a)-(c), a notable concurrence is observed between the semi-analytical and numerical outcomes, represented by solid circles. It is evident that for a given $Le$ value, both the convective strength $\Psi_0$, $Nu$, and $Sh$ increase as the power-law index, $n$, decreases. For $Le < 1$, the mass transfer predominates the convective flow and accentuates...
the heat transfer. Conversely, for Le > 1, the convective strength, along with the heat and mass transfer rates, augments with Le, eventually leveling off with a high Le value. The flow's rotational direction is directly contingent on the Lewis number's value. As depicted in Figures 11(a) and 11(d), the flow rotates clockwise (Ψ₀ < 0) when Le < 1 and counterclockwise (Ψ₀ > 0) when Le > 1. To elucidate the flow rotation's direction, let's examine the case with Le > 1. In such a scenario, where the thermal diffusivity, α, surpasses the mass diffusivity, D, both analytical and numerical solutions concur that the flow's circulation is counterclockwise. With α > D, the volume of mass transported by convection along the vertical cavity walls outweighs the heat's volume. Consequently, the quantity of heat transferred through the system supersedes that of mass. This creates a dominant horizontal temperature gradient. As thermal effects contribute to a density decrease on the right side of the cavity and a density increase on the left side, a counterclockwise flow circulation is induced.

Figure 11. Effect of Lewis number Le and power-law index n on (a) the flow intensity Ψ₀, (b) the Nusselt number Nu, and (c) Sherwood number Sh.

![Graphs showing the effect of Lewis number and power-law index on flow intensity, Nusselt number, and Sherwood number.](source: Authors)
Table 4. Dependence of $Le_{sub}$ on $Le$ for $\Phi=90^\circ$, $R_T=100$, and $N=-1$.

<table>
<thead>
<tr>
<th>$Le$</th>
<th>$Le_{sub}^&gt;$</th>
<th>$Le_{sub}^&gt;$</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.87</td>
<td>0.88</td>
<td>0.83</td>
<td>0.80</td>
<td>0.77</td>
</tr>
<tr>
<td>0.8</td>
<td>1.12</td>
<td>1.15</td>
<td>1.17</td>
<td>1.190</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Source: Authors

6 CONCLUSION

This paper presents a semi-analytical and numerical investigation of double-diffusive convection in an inclined porous layer saturated by non-Newtonian power-law fluid and heated and salted from the bottom. The analysis considers the impact of the thermal Rayleigh number $R_T$, buoyancy ratio $N$, Lewis number $Le$, power-law index $n$, and angle of inclination $\Phi$ on the intensity of convection and heat and mass transfer, across a broad range of governing parameters.
The main findings derived from the current analysis are outlined as follows:

(a) In the scenario where enclosures exhibit a long aspect ratio $A >> 1$, the parallel flow approximation has been utilized to simplify the initial set of governing partial differential equations, resulting in a notable alignment between the numerical and asymptotic solutions. While the derived semi-analytical model still necessitates a numerical process, this is significantly less demanding than numerically solving the original full set of governing equations.

(b) The study showed a direct correspondence between diminishing the power-law index, $n$, and increasing both the strength of convection $\Psi_0$ and the resultant rates of heat $Nu$ and mass $Sh$ transfer.

(c) The spatial orientation of the enclosure significantly influences convective heat and mass transfer for specific values of the Rayleigh number $R_T$ and the power-law index $n$. Irrespective of the chosen power-law index, $n$, optimal heat and mass transfer rates are observed when the layer is successively heated from the bottom within the angular range of $(0^\circ < \Phi < 90^\circ)$ and $(60^\circ < \Phi < 120^\circ)$. The Nusselt number peaks, associated with a given Rayleigh number, are contingent upon certain angles, denoted as $\phi_{\text{max}}$, with their values varying notably with the power-law index, $n$. As an illustration, for $R_T = 50$, the corresponding $\phi_{\text{max}}$ is $40^\circ$ when $n = 1.2$ and $36^\circ$ when $n = 0.8$.

(d) It was noticed that for an inclined cavity with $\Phi = 90^\circ$ (i.e., Vertical cavity), the Nusselt number $Nu$ and the Sherwood number $Sh$ tend to asymptotic constant values for specific $n$. These constants are exclusively determined by the power-law index $n$, regardless of the values of $N$ and $Le$.

(e) For Lewis values less than 1, mass transfer dominates the convective flow, causing a clockwise rotation. Conversely, for $Le > 1$, the flow rotates counterclockwise, and with high $Le$ values, the convection strength, along with heat and mass transfer rates, tends to be constant.

(f) For buoyancy ratios $N$ below -1, the transition zone between thermally driven single-cell rotation and solute-driven single-cell rotation is highly reliant on the power-law index $n$ of the non-Newtonian fluid.
Comprehending double-diffusive convection in porous media, particularly with non-Newtonian fluids, holds considerable importance for both societal and academic realms. This understanding finds practical applications across diverse industries like solar energy systems, nuclear reactors, and industrial processes, offering avenues for enhancing energy efficiency and reduced environmental impact.

The research outcomes have the potential to enrich academia by pushing the boundaries of comprehension regarding intricate fluid dynamics phenomena. Moreover, they can play a pivotal role in refining mathematical models and computational methodologies, thereby facilitating more precise simulations of natural convection across various scenarios.

Moreover, the methodologies utilized in this study, comprising numerical and semi-analytical solutions, provide valuable instruments for forthcoming inquiries in related domains. Researchers can leverage these approaches to delve into supplementary parameters, enhance prevailing models, and tackle novel research inquiries. In essence, the outcomes derived from this research hold promise for catalyzing advancements in both practical applications and theoretical comprehension across the scientific community.

In this investigation, the mathematical model relies on several simplifications. In practical scenarios, porous media exhibit anisotropic behavior, and fluid characteristics deviate from Newtonian assumptions. To advance our comprehension, it would be valuable to:

- reexamine this investigation using a non-Newtonian model, accounting for porous media anisotropy with varying permeability and thermal conductivity.
- Exploring alternative geometric configurations beyond those explored herein would also be insightful.
- Moreover, extending the model to encompass three-dimensional analyses, accommodating intricate geometries and diverse boundary conditions, is suitable.

A pivotal goal, necessitating substantial effort, is to validate the attained outcomes through experimental validation.
REFERENCES


ABBREVIATIONS

\( A \) aspect ratio of the cavity, \( L'/H' \)
\( C_s \) dimensionless concentration gradient in \( x \)-direction, Eq. (30)
\( C_r \) dimensionless temperature gradient in \( x \)-direction, Eq. (30)
\( D \) molecular diffusion coefficient, \( m^2/s \)
\( g \) gravitational acceleration, \( m/s^2 \)
\( h \) consistency index, \( Pa \cdot s^n \)
\( H' \) thickness of enclosure, \( m \)
\( j' \) constant solute flux per unit area, \( kg/(ms) \), Eq. (5)
\( k \) thermal conductivity, \( W/(mK) \)
\( K \) permeability of porous medium, \( m^2 \)
\( L' \) length of cavity, \( m \)
\( Le \) Lewis number, \( a/D \)
\( n \) power-law index
\( N \) buoyancy ratio, \( \beta'\Delta S'/\beta_0'\Delta T' \)
\( Nu \) Nusselt number, Eq. (19)
\( p' \) pressure
\( q' \) constant heat flux per unit area, \( W/m^2 \), Eq. (4)
\( R_T \) thermal Rayleigh number, \( K\rho_0 g\beta_0'\Delta T' (H'/\alpha)^n / \varepsilon \)
\( S \) dimensionless concentration
\( \Delta S' \) characteristic concentration difference, \( j'/\rho D \)
\( Sh \) Sherwood number, Eq. (19)
\( T \) dimensionless temperature
\( \Delta T' \) characteristic temperature difference, \( q'L'/k \)
\( \bar{v} \) superficial velocity, Eq. (1)
\( u \) dimensionless velocity \( x \)-component
\( v \) dimensionless velocity \( y \)-component
\( x, y \) cartesian coordinates

Greeks symbols

\( \alpha \) fluid thermal diffusivity, \( m^2/s \)
\( \beta' \) solute expansion coefficient, Eq. (7)
\( \beta''_r \) thermal expansion coefficient, Eq. (7)
\( \rho \) density of fluid, \( kg/m^3 \)
\( \mu' \) apparent viscosity in power-law model, \( Pa \cdot s^n \), Eq. (2)
\( \varepsilon \) parameter in power-law model, \( Pa \cdot s^n / m^2 \), Eq. (3)
\( \phi \) porosity of the porous medium
\( \sigma \) heat capacity ratio, \( (qC)_p/(qC)_f \)
\( \xi \) normalized porosity of the porous medium, \( \xi = \phi / \sigma \)
\( \Psi \) dimensionless stream function, \( \Psi / \alpha \)
\( \Phi \) inclination angle of the cavity

Superscript

\( ' \) dimensional quantities

Operators

\( J(f, g) = \frac{\partial f}{\partial y} \frac{\partial g}{\partial x} - \frac{\partial f}{\partial x} \frac{\partial g}{\partial y} \)

Subscript

\( max \) maximum value
\( 0 \) condition at the origin of the coordinate system
\( o \) refers to the value taken at the centre of the cavity