Model order reduction, a novel method using krylov sub-spaces and genetic algorithm

Redução da ordem do modelo, um novo método usando subespaços de Krylov e algoritmo genético

DOI: 10.54021/seesv5n1-030
Recebimento dos originais: 19/02/2024
Aceitação para publicação: 08/03/2024

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ABSTRACT
Model Order Reduction (MOR) of complex and large systems in Electrical engineering, continuous to be an attractive field for Engineers and Scientists over the last few decades, this complexity of models makes the control designs and simulation using Computer Aided Design (CAD) more and more difficult and consuming a lot of time. There for, accurate, robust and fast algorithms for simulation are needed. The goal of MOR is to replace the original system by an appropriate reduced system which preserves the main properties of the original one such that stability and passivity. Several analytical MOR techniques have been proposed in the literature over the past few decades, to approximate high order linear dynamic systems like Krylov sub-space techniques and SVD (Singular Value Decomposition) techniques. However, most of these techniques lead to computationally demanding, time consuming, iterative procedures that usually
result in non-robustly stable models with poor frequency response resemblance to the original high order model in some frequency ranges. Recently a set of new techniques based on Artificial Intelligence (AI) were proposed in [1] for MOR. This article considers the problem of model order reduction of Linear Time In varying (LTI) systems. It is described by first and second order ordinary differential equations model. A tow steps method for model order reduction of LTI systems is proposed here, which combined features of an analytic technique (Krylov approach) and an AI technique (Genetic Algorithm). In the first step, the size of the original model is reduced to an intermediate order, using an analytical technique based on Krylov sub-spaces. In the final step of the reduction process, an AI approach based on Genetic Algorithm (GA) is applied to obtain an optimized nominal model.

**Keywords**: Krylov subspace, model order reduction, LTI systems, second order system, genetic algorithm, state space, transfer function.

**RESUMO**

A redução da ordem de modelos (MOR) de sistemas grandes e complexos em engenharia elétrica continua sendo um campo atraente para engenheiros e cientistas nas últimas décadas. Essa complexidade dos modelos torna os projetos de controle e a simulação usando o projeto auxiliado por computador (CAD) cada vez mais difíceis e consomem muito tempo. Por isso, são necessários algoritmos precisos, robustos e rápidos para a simulação. O objetivo do MOR é substituir o sistema original por um sistema reduzido adequado que preserve as principais propriedades do sistema original, como estabilidade e passividade. Nas últimas décadas, várias técnicas analíticas de MOR foram propostas na literatura para aproximar sistemas dinâmicos lineares de alta ordem, como as técnicas de subespaço de Krylov e as técnicas de SVD (Decomposição de Valor Singular). No entanto, a maioria dessas técnicas leva a procedimentos iterativos demorados e exigentes do ponto de vista computacional que, em geral, resultam em modelos não robustos e estáveis, com baixa semelhança de resposta em frequência com o modelo original de alta ordem em algumas faixas de frequência. Recentemente, um conjunto de novas técnicas baseadas em Inteligência Artificial (IA) foi proposto em [1] para MOR. Este artigo considera o problema da redução da ordem do modelo de sistemas LTI (Linear Time In varying). Ele é descrito por um modelo de equações diferenciais ordinárias de primeira e segunda ordem. Um método de duas etapas para a redução da ordem do modelo de sistemas LTI é proposto aqui, combinando recursos de uma técnica analítica (abordagem de Krylov) e uma técnica de IA (algoritmo genético). Na primeira etapa, o tamanho do modelo original é reduzido a uma ordem intermediária, usando uma técnica analítica baseada em subespaços de Krylov. Na etapa final do processo de redução, uma abordagem de IA baseada em Algoritmo Genético (AG) é aplicada para obter um modelo nominal otimizado.

**Palavras-chave**: subespaço de Krylov, redução da ordem do modelo, sistemas LTI, sistema de segunda ordem, algoritmo genético, espaço de estado, função de transferência.
1 INTRODUCTION

Compact modeling of passive RLC interconnected networks has been a research-intensive area in the past decade due to increasing adverse deep submicron effect and interconnects–dominant delays in current high performance VLSI design [2, 3]. A number of projections-based Model Order Reduction (MOR) techniques have been introduced [4-7] to analyze the transient behavior of interconnect.

In order to carry on the analysis, synthesis and design of VLSI circuits or any Electrical system, we began by the development of a “Mathematical Model” that can be substituted for the real system. This mathematical description can take different forms such as a set of Partial Differential Equations (PDEs) or discrete Ordinary Differential Equations (ODEs) depending on the system as well as the approach used for modeling [8-11]. In particular, the numerical tools like Finite Element (FE) and Finite Difference (FD) methods have become very popular for modeling and analysis of such systems. After discrediting the PDEs through an FE procedure, a model consisting of large number of ODEs is obtained. It is difficult to work with such a large model; hence, a simplification of the model is a must to make the model useful. This simplification of the model is known as Model Order Reduction (MOR) i.e. obtaining the low order model of existing high order model such that both are equivalent in terms of response. Here we consider the system in the form of Transfer Function (TF) and the State Space (SS) to establish a Reduced-Order Model (ROM). Further a numerous methods of MOR are also available in the literature [12-17], which are based on minimization of the Integral Square Error (ISE) criterion.

A leading method in MOR is moment matching by means of Krylov Sub-space, by approximating a certain number of moments of the original system transfer function. But, the accuracy of the output response of this reduced system in time domain cannot always be guaranteed even if the reduced transfer function can be accurate in frequency domain.

The main objective of the present work is to obtain a reduced model order of a second order system using a novel reduction technique proposed here. It involves a combination of Krylov sub-space technique and a Genetic Algorithm (GA). this a further extension of the Artificial Intelligence approaches recently
proposed in [1] which combines Krylov Sub-spaces technique and a GA. The numerical results will prove the superiority and the advantage of our work over the conventional algorithms and the GA proposed in [1].

This article is organized as follows: in Section 2 we have presented the modeling approaches (TF and SS), Krylov Sub-space for MOR and GA are discussed in Section 3. A new reduction technique for second order system which combined both Krylov Sub-space and GA is proposed in Section 4. Numerical results are shown in Section 5. Finally, conclusions are given in Section 6. The main aim of this study is to use a novel technique which we called Krylov-GA technique to solve complex model reduction problems, and help obtain globally optimized reduced order models.

2 MODELING
2.1 LINEAR TIME-INVARIANT STATE SPACE MODELS

A continuous time-invariant linear (LTI) dynamical system is of the form

\[
\begin{align*}
    x(t) &= Ax(t) + Bu(t) \\
    y(t) &= Cx(t)
\end{align*}
\]

(1)

With the initial condition \( x(0) = x_0 \).

Here \( t \) is the time variable,
\( x(t) \in \mathbb{R}^N \) is state vector,
\( u(t) \in \mathbb{R}^m \) the input excitation vector, and
\( y(t) \in \mathbb{R}^p \) the output measurement vector,
\( A \in \mathbb{R}^{N \times N} \) is the system matrix, \( B \in \mathbb{R}^{N \times m} \) and
\( C \in \mathbb{R}^{N \times p} \) are respectively the input matrix and the output matrix,
\( N \) is the state space dimension and \( m \) and \( p \) are the number of inputs and outputs, respectively.

In the most practical cases, we can assume that \( m \) and \( p \) are much smaller than \( N \) and \( m \geq p \).

Linear systems arise in many applications, such as the Network circuits with linear elements [9], structural dynamics analysis with lapped mass and stiffness elements [18-19], linearization of a nonlinear system around equilibrium points [20].

The linear systems of the form (1) are often referred to as the representation of the system in the time domain, or in the state space. Equivalently, by taking the Laplace transformation defined by:
\[ F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt, \quad s \in \mathbb{C} \] \hfill (2)

And assuming zero initial conditions, we obtain a representation of (1) in the frequency domain known as the transfer function \( G(s) \) satisfying:

\[
G(s) = C(sI - A)^{-1}B \\
Y(s) = G(s)U(s)
\] \hfill (3)

The following types of analysis are typically performed for a given linear dynamical system of the form (1) [13]:

- **Static (DC) analysis**: to find the point which the system settles in the equilibrium, or rest.
- **Steady-state analysis**, also called frequency response analysis, to determine the frequency response \( G(s) \) of the system to external steady-state oscillatory (i.e. sinusoidal) excitation.
- **Model frequency analysis**: to find system natural vibrating frequency modes and their corresponding modal shapes.
- **Mean Square Error (MSE)**: to measure the average of the square of the error between the responses of the original and reduced systems.
- **Transient analysis**: to compute the output behavior \( y(t) \) subject to time-variant excitation \( u(t) \).

In this paper we will focus on applying reduction-order modeling techniques for Mean Square Error (MSE) only.

### 2.2 SECOND ORDER LTI SYSTEMS

In general, second-order linear time-invariant dynamical systems always arise in RLC circuits [21], Micro electromechanical systems (MEMS) [22] and structure engineering [23], given by

\[
\begin{align*}
(M\ddot{x}(t) + D\dot{x}(t) + Kx(t)) = Gu(t) \\
y = Lx(t)
\end{align*}
\] \hfill (4)

Where \( M, D, \) and \( K \) are the usual mass, damping, and stiffness matrices and \( x \) is the vector of generalized displacements. The matrix \( G \) distributes the force input \( u \) to the correct degrees of
freedom. Typically not all of the degrees of freedom are measured, and so the measured displacement $y$ will be related to $x$ by $Lx(t)$.

With $n$ second order differential equations, $m$ inputs and $p$ outputs. Equivalently, the model (4) can be written in state space with $N = 2n$ first order differential equations as follows,

$$
E \dot{z}(t) = Az(t) + Bu(t)
$$

(5)

$$
y(t) = Cz(t)
$$

(6)

Where the state vector $z$ is

$$
z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}
$$

(7)

And $E$, $A$, $B$ and $C$ are defined as

$$
A = \begin{bmatrix} 0 & F \\ -K & -D \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ G \end{bmatrix},
$$

$$
E = \begin{bmatrix} F \\ 0 \\ M \end{bmatrix}, \quad C = [L \ 0],
$$

(7)

Where $F \in \mathbb{R}^{n \times n}$ is a nonsingular matrix. For simplicity, $F = I$ can be chosen or for the case that $K$ is singular $F = K$.

3 MODEL ORDER REDUCTION

3.1 KRYLOV SUB-SPACES FOR MOR

The most Model Order Reduction procedures are accomplished by means of projection, the objective is to approximate the state vector $x(t)$ to a low-dimensional sub-space. This is achieved by the substitution

$$
x(t) = V x_r(t) + \epsilon(t)
$$

(8)

Where $V \in \mathbb{R}^{N \times q}$ is the transformation matrix, $x_r(t) \in \mathbb{R}^{q}$ the reduced state vector, $\epsilon(t)$ the residual and $q \ll N$, inserting the projection equation (8) into the linear system in Eq (1) results in a low-dimensional approximation

$$
V \dot{x}_r(t) = AV x_r(t) + BV u(t) + \epsilon(t)(t),
$$

(9)

$$
y(t) = CV x_r(t)
$$
In general, this system is over determined having \( q \) unknowns but \( n \) equations. To solve this problem and find a unique solution, Eq.(9) is pre-multiplied by second transformation matrix \( W \in \mathbb{R}^{n \times q} \) such \( W^T \varepsilon(t) = 0 \), the residual equal to zero and

\[
W^T V \dot{x}_r(t) = W^T A V x_r(t) + W^T B u(t), \quad y(t) = C V x_r(t)
\]  

(10)

Where \( A_r = W^T A V \), \( B_r = W^T B \) and \( C_r = C V \). The system in Eq.(6) is called general reduced order model (ROM) by projection. The number of inputs \( u(t) \) and outputs \( y(t) \) remains the same thought the order of the system.

For most of the methods, the aim of model order reduction is to provide the projection matrices \( W \) and \( V \) such that their calculation is computationally efficient as well as automatable and the systems characteristics are preserved. The Krylov sub-space based model reduction methods have been developed in order to produce reduced order models of large-scale linear systems efficiently and stably via projection onto subspaces that satisfy specific conditions. These conditions are based on requiring the reduced order transfer function to much selected moments of the transfer function \( G(s) \) of the original system.

**Definition 1**: Let \( M \in \mathbb{C}^{n \times m} \) and \( X \in \mathbb{C}^{n \times m} \). A Krylov subspace of order \( k \) of the pair \((M,X)\), denoted \( K_k(M,X) \), is the image of the matrix \([M \ M X \ ... \ M^{k-1}X]\).

Considering the transfer function in Eq. (3) for MIMO case, the expansion of \( G(s) \) around infinity gives

\[
G(s) = C(sI - A)^{-1}B = \sum_{i=0}^{\infty} R_i^\infty s^{-i-1}
\]  

(11)
Where the coefficients $R_i^\infty$ are called the Markov parameters of the system or the moments of the transfer function. One intuitive way to approximate $G(s)$ is to construct a transfer function, $\hat{G}(s)$.

$$\hat{G}(s) = \hat{C}(sI_n - \hat{A})^{-1}\hat{B} = \sum_{i=1}^{\infty} \hat{R}_i^\infty s^{-i}$$  \hspace{1cm} (12)

Such that $\hat{R}_i^\infty = R_i^\infty$ for $1 \leq i \leq$, where $r$ is as large as possible and is generically equal to $2n$. The resulting transfer function $\hat{G}(s)$ generally approximates quite well the original transfer function for large values of $s$.

Depending on the expansion point, the moments and its corresponding MOR schemes are defined as:

- Expansion point $s=0$ results in a MacLaurin series. The corresponding MOR procedure is called Padé-Approximation.
- Expansion point $s = s_k$ results in a Taylor series. The corresponding MOR procedure is called Shifted Padé-Approximation, Rational Interpolation or Multi-point Padé-approximation.
- Expansion point $s \to \infty$ results in a Markov series. The corresponding MOR procedure is called Partial Realization.

<table>
<thead>
<tr>
<th>Table 1 - Characteristics of Krylov subspace methods</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Advantages</strong></td>
</tr>
<tr>
<td>• Iterative method</td>
</tr>
<tr>
<td>• Easy to implement</td>
</tr>
<tr>
<td>• Suitable for very large scale systems</td>
</tr>
<tr>
<td>• Computational efficient</td>
</tr>
<tr>
<td>• Stationary exact</td>
</tr>
<tr>
<td>• Robust calculation</td>
</tr>
</tbody>
</table>

Source: Authors.

The sub space $K_k(A^{-1}, AB)$ is called input Krylov sub space for the system (1), this sub space is used for the purpose of reduction by the mean of Arnoldi process.

**Theorem 1**: if the matrix $V$ used in (6), is a basis of the Krylov sub space $K_k(A^{-1}A^{-1}B)$ and the matrix $W$ is chosen such that the matrix $A$, is non singular, then the first $k$ moments (around zero) of the original and the reduced systems match [24-25].

**Algorithm 1.** Krylov subspace (Kr)
Start: set $r = A^{-1}B$, $v_1 = \frac{r}{\|r\|_2}$.

(1) For $k=1,2,3,\ldots$ Do,

(a) Calculating the next vector

\[ V_{k+1} = A^{-1}v_k. \]

(b) Orthogonalization: for $j = 1$ to $I$,

\[ H_{jk} = v_j^T v_{k+1}. \]

\[ v_{k+1} = v_{i+1} - H_{jk} v_j. \]

\[ H_{k+1,k} = \|v_{k+1}\|_2. \]

(c) Normalization: if $H_{k+1} = 0$, break the loop, otherwise:

\[ v_{k+1} = \frac{v_{k+1}}{H_{k+1,k}}. \]

(d) Increase $k$ and go to (1.a).

3.2 GENETIC ALGORITHMS

In recent years, one of the most promising research fields has been “Evolutionary Techniques”, an area utilizing analogies with nature or social systems. Evolutionary techniques are finding popularity within research community as design tools and problem solvers because of their versatility and ability to optimize in complex multimodal search spaces applied to non-differentiable objective functions. Genetic Algorithms are a family of Artificial Intelligence techniques inspired by evolution. These algorithms encoded a potential solution to a specific problem on a simple chromosome-like data structure and apply recombination operators to these structures to preserve critical information. GAs are often viewed as function optimizers, although the range of problems to which GAs have been applied in quiet broad. Genetic Algorithms are composed of three main operators:

- **Reproduction**: is the process in which individual strings are copied according to their fitness function’s value.
• **Crossover**: is the process in which members of the newly reproduced strings in the mating pool are mated at random.

• **Mutation**: is the occasional random alteration of the value of a string position.

Implementation of GA requires the determination of six fundamental issues: chromosome representation, selection function, the genetic operators, initialization, termination and evaluation. Brief descriptions about these issues are provided in the following sections:

A. **Chromosome representation**: chromosome representation scheme determines how the problem is structured in the GA and determines the genetic operators that are used. Each individual or chromosome is made up of a Sequence of genes. Various types of representations of an individual or chromosome. Generally, natural representations are more efficient and produce better solutions.

B. **Selection function**: to produce successive generations, selection of individuals plays a very significant role in a genetic algorithm. The selection function determines which of the individuals will survive and move on to the next generation. A probabilistic selection is performed based upon the individual's fitness such that the superior individuals have more chance of being selected. There are several schemes for the selection process: roulette wheel selection and its extensions, scaling techniques.

C. **Genetic operators**: the genetic operators provide the basic search mechanism of the GA. There are two basic types of operators: crossover and mutation. These operators are used to produce new solutions in the population. Crossover takes two individuals to be parents and produces two new individuals while mutation alters one individual to produce a single new solution.

D. **Initialization, termination and evaluation function**: an initial population is needed to start the genetic algorithm procedure. The initial population can be randomly generated or can be taken from other methods. The Genetic Algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. If the algorithm has terminated due to a maximum number of
generations, a satisfactory solution may or may not have been reached [26-27]. Other useful references on GAs and their applications are included in [28-31].

The fitness function is the common evaluation factor among all selection techniques. Fitness function is a function, which evaluates the fitness of each individual to be selected for the next generation. The fitness function is always a measure of error for the step response of the reduced order model and the original model. An example for the error calculation fitness function is the mean square error, which can be calculated as follows:

\[ f = \frac{1}{n} \sum_{i=1}^{n} f_i \]  

(13)

The term \( f_i \) is calculated as the square of difference between the original step response and the reduced transfer function step response.

Figure 2 - The GA Flow chart

- **Initial population**
- **End Criterion Reached?**
  - **Yes**
  - **No**
- **Selection**
- **Cross over**
- **Mutation**
- **New population**

Source: Authors.
4 PROPOSED METHOD

The proposed methodology is applied to a LTI system of \( n \)th order of \( q \) inputs and \( r \) outputs described by state space in time domain as shown in eq.(1) for the 1st order system and eq.(5) for 2nd order system. By converting the state space model from time domain to frequency domain, the transfer function TF can be described in eq.(14):

\[
G(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{n-1} A_i s^i}{\sum_{i=0}^{n} a_i s^i}
\]  

(14)

Where \( N(s) \) is the numerator and \( D(s) \) is the denominator of the higher order, \( A_i \) and \( a_i \) are the coefficients on numerator and denominator respectively. The problem is to find an \( m \)th lower order model \( R_m(s) \), where \( m < n \) in the following form:

\[
R(s) = \frac{N_m(s)}{D_m(s)} = \frac{\sum_{i=0}^{m-1} B_i s^i}{\sum_{i=0}^{m} b_i s^i}
\]  

(15)

In addition, we must take into account that the reduced model preserves the characteristics of the original transfer function.

Figure 3 - KRYLOV-GA MOR Model

Source: Authors
Our proposed approach in 6 inputs and 4 outputs, where at the 1st stage, the original state space model and intermediate order \( k \) are used to obtain a reduced state space model of order \( k \) by Krylov method, which converted to a transfer function form to be an input for the 2nd stage of reduction process with the other inputs (Number of generation, Iterations, and the final desired order). The outputs are the reduced model transfer function, the mean square error; the time elapsed for reducing the transfer function, and the step and frequency responses of the models. Fig.2 shows the diagram of the proposed method.

5 NUMERICAL TESTS

Tow numerical examples are chosen from [1] for the comparison of reduced system using the proposed method and the reduced system from [1] and the original system. The simulations are carried on MatlabR2011a software, core i5, with 8GB Ram memory. The results of the testing procedures are: A step response diagram, the frequency response using Bode diagram, MSE, the time elapsed. The tow original transfer functions used for simulation are:

\[
G_1(s) = \frac{s^9+59s^8+1657.9s^7+28640.7s^6+334474.5s^5+2742122.s^4+15867513.2s^3+62854022.s^2+155453416.s+182004673.4}{s^{10}+49.5s^9+1168.6s^8+17109.s^7+171437.4s^6+1227121.s^5+6376179.3s^4+24002493.s^3+64135934.s^2+114485916.5s+110910010.4}
\]  

(16)

\[
G_2(s) = \frac{s^9+46.8s^8+957.6s^7+11144.s^6+80511..s^5+369601.6s^4+106074.5s^3+1809006.4s^2+1669555.4s+638266}{s^{10}+36.9s^9+620.8s^8+6257.9s^7+41888.s^6+195879.7s^5+658023.2s^4+1611073.5s^3+2857356.s^2+3425885.4s+2110138.4}
\]  

(17)

Both the transfer functions are transformed to state space model using the MATLAB command \texttt{tf2ss} to be an input to our program. The reduced models from [1] are:

\[
R_1(s) = \frac{1.167 s+16.69}{s^2+2.035 s+10.21}
\]  

(18)

\[
R_2(s) = \frac{2.574 s+1.847}{s^2+0.707 s+6.238}
\]  

(19)

The reduced 2nd order models from the proposed method are:
\[ KR_1(s) = \frac{0.8738s + 17.62}{s^2 + 2.102s + 10.81} \]  \hspace{1cm} (20)

\[ KR_2(s) = \frac{2.501s + 1.951}{s^2 + 0.6414s + 6.308} \]  \hspace{1cm} (21)

A comparison has been made between the results of the new proposed model and the results from [1], our proposed model showed its superiority and efficiency in terms of accuracy over the model [1]. The 1\textsuperscript{st} comparison was made between \( G_1(s), R_1(s) \) and \( KR_1(s) \) in terms of step response as shown in Fig.3; while the frequency responses are shown in Fig.4, the results in term of Step responses MSE of our model and the mathematical methods are compared in table.2.

Figure 4 - Step response for Original Model G1 compared to RK1 and R1

Figure 5 - Frequency response for Original Model G1 compared to RK1 and R1
The second comparison is carried on $G_2(s)$, $R_2(s)$ and $KR_2(s)$, Figure 5 and Figure 6 shows the step and frequency responses respectively.

Figure 6 - Step response for Original Model $G_2$ compared to RK2 and R2

![Step response graph](image)

Source: Authors.

Figure 7 - Frequency response for Original Model $G_2$ compared to RK2 and R2

![Frequency response graph](image)

Source: Authors.

Table 2. A comparison in term of Step responses MSE for all techniques for the $G_1(s)$ and $G_2(s)$.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Krylov-GA</th>
<th>GA</th>
<th>Pade</th>
<th>Routh</th>
<th>Balanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step response MSE for $G_1(s)$ %</td>
<td>0.055</td>
<td>0.07</td>
<td>176</td>
<td>5.28</td>
<td>0.05</td>
</tr>
<tr>
<td>Step response MSE for $G_1(s)$ %</td>
<td>0.055</td>
<td>0.07</td>
<td>176</td>
<td>5.28</td>
<td>0.05</td>
</tr>
<tr>
<td>Time</td>
<td>5 sec</td>
<td>23 sec</td>
<td>0.1 sec</td>
<td>0.1 sec</td>
<td>0.1 sec</td>
</tr>
</tbody>
</table>

Source: Authors.
The numerical test of the proposed model demonstrate the efficiency of this new Krylov-GA approach over the pure GA-based MOR and also over the main mathematical methods in terms of accuracy and preservation of the proprieties of the original system. Krylov-GA MOR is a promising technique in reducing large-scale linear systems both 1st and 2nd order systems.

6 CONCLUSION

The detailed mathematical modeling of modern engineering systems leads to high order dynamic systems. For simplicity of simulation, interpretation, and control of such processes it is desirable to represent the dynamics of these high order systems by lower order models. In this paper, we propose a new MOR method for first and second order linear systems based on Krylov subspace technique combined with Genetic Algorithm. Using this reduction technique, output of both the original and reduced models are almost same, but the size of the reduced model is further reduced using KRYLOV-GA technique to make it more suitable for control applications. The proposed methodology showed superiority in terms of accuracy, time consuming elapsed by the MOR process, and preserving the basic properties of the original complex model over the pure GA method. The results lead to summarize in the following contributions:

1. This work is first to solve the MOR problems using a new technique based on both analytic and AI techniques
2. A computationally attractive and analytically simple model reduction approach based on Krylov sub-space and Genetic Algorithm is introduced.
3. Improved reduced order models are obtained for benchmark model reduction problems.

Some possible future research area in model reduction cloud be a modified Krylov-GA algorithm to take MIMO systems and the study the performances of Krylov-GA algorithm in reducing MIMO systems. Another possible research area would be to study an SVD-GA technique in model reduction.
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