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Fuzzy predictive controller for trajectory tracking of a wheeled mobile robot

Controlador preditivo fuzzy para rastreamento de trajetória de uma raiz móvel com rodas

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ABSTRACT
This paper presents a mobile robot control methodology that uses a fuzzy predictive control system to accurately track given paths. It is specifically designed for scenarios of two successive and distinct paths within a shared reference trajectory. Through the combination of fuzzy logic and predictive control methods, the system aims to significantly enhance tracking accuracy. Modelling the system, using a T-S fuzzy system provides a comprehensive model framework to optimize the tracking process. Furthermore, the use of a well-tuned fuzzy system facilitates dynamic adjustments of the weighting matrices of the predictive controller. The combination of fuzzy logic and predictive techniques results in a robust control system capable of handling complex tracking tasks. The simulation results describe the accuracy, robustness, and efficiency of the suggested control strategy. The system is particularly effective in scenarios with two successive paths within a shared reference trajectory, where precise tracking is essential. This approach is crucial for mobile robots or vehicles navigating complex, changing environments.

Keywords: Model predictive control, nonholonomic mobile robot, T-S Fuzzy logic, Fuzzy predictive control, tracking error.

RESUMO
Este artigo apresenta uma metodologia de controle de robô móvel que usa um sistema de controle preditivo difuso para rastrear com precisão determinados caminhos. Ele foi projetado especificamente para cenários de dois caminhos sucessivos e distintos em uma trajetória de referência compartilhada. Por meio da combinação de lógica difusa e métodos de controle preditivo, o sistema visa a aumentar significativamente a precisão do rastreamento. A modelagem do sistema, usando um sistema difuso T-S, fornece uma estrutura de modelo abrangente para otimizar o processo de rastreamento. Além disso, o uso de um sistema difuso bem ajustado facilita os ajustes dinâmicos das matrizes de ponderação do controlador preditivo. A combinação de lógica difusa e técnicas preditivas resulta em um sistema de controle robusto capaz de lidar com tarefas de rastreamento complexas. Os resultados da simulação descrevem a precisão, a robustez e a eficiência da estratégia de controle sugerida. O sistema é particularmente eficaz em cenários com dois caminhos sucessivos dentro de uma trajetória de referência compartilhada, em que o rastreamento preciso é essencial. Essa abordagem é fundamental para robôs ou veículos móveis que navegam em ambientes complexos e mutáveis.

Palavras-chave: controle preditivo de modelo, robô móvel não holonômico, lógica Fuzzy T-S, controle preditivo Fuzzy, erro de rastreamento.

1 INTRODUCTION

Over the last several years, Autonomous vehicles and mobile robots have experienced significant development due to the growing need to carry out a range of tasks across industries, the military sector, healthcare, and transportation On the other hand, recent advances in modern control techniques have played a crucial role in addressing various challenges associated with mobile robot functionality, particularly in trajectory tracking, which integrates sensing and control for navigation in complex environments and reaching the target destination [1].

Due to the need for accurate path tracking, researchers have developed pioneering techniques in both predictive control and fuzzy control, offering numerous benefits and accomplishments in this domain. In fact, the authors of [2] introduced ground-breaking research on applying MPC to track the trajectory of mobile robots. This approach employed an explicit optimal controller based on a linearized kinematic model, pre-computed offline, and rigorously validated through simulations and experiments. Expanding upon this foundation, [3] conducted a comprehensive review of the utilization of MPC in motion control of wheeled mobile robots. This encompassed a diverse range of MPC models, various robot kinematic setups, and a multitude of motion tasks. The study also demonstrated real-time implementation through experimental scenarios, providing valuable information for future research tasks.

The researchers in [4], introduced an innovative approach, employing an optimal fuzzy logic controller based on Mamdani-type to track the trajectory of a mobile robot on wheels. The proposed controller dealt with uncertainties by concurrently optimizing the coefficients of the PID controller and the membership functions employing the Particle Swarm Optimization algorithm with a random inertia weight, resulting in positive simulation results. Subsequently, in [5], authors proposed a MPC strategy for trajectory tracking of a mobile robot. This strategy focused on controlling the voltage of the drive motors while taking into account various constraints.

In another direction, authors of [6] presented a refined model predictive control strategy for trajectory tracking of a differential drive robot. This approach combined linearized dynamic and kinematic models, coupled with controlled motor voltages, and was substantiated by compelling simulation results. Further enhancement was made in [7] where the authors introduced a continuous form of the predictive controller based on the tracking error model. This continuous form exhibited superior performance in both tracking accuracy and control effort compared to its discrete form. Moreover, the authors of [8], introduced a novel behaviour-based fuzzy controller for a nonholonomic mobile robot with a differential drive. They showed, through simulations and experiments, the controller's effectiveness in achieving target navigation while successfully avoiding obstacles in a dynamic environment.

In [9], a novel technique named tracking-oriented model predictive static programming was introduced. This approach was specifically designed for efficient trajectory tracking of a two-wheel differential drive mobile robot. The technique's efficiency was demonstrated through a combination of simulations and real hardware experiments, tackling a spectrum of real-world challenges. Simultaneously, [10] unveiled an integrated control strategy. These latter combined kinematic with dynamic models, incorporating adaptive fuzzy integral terminal sliding mode control and robust compensation. By addressing parameter uncertainties and disturbances, this approach was effectively validated through simulations for the purpose of tracking trajectories in nonholonomic wheeled mobile robots. To further broaden the scope of applicability, [11] focused on the development of a specialized controller for fast rovers with independent steering. This controller harnessed a dynamic model and Non-linear Continuous-time Generalized Predictive Control to achieve precise and robust path tracking in dynamic environments.

In [12], researchers introduced a robust fuzzy logic proportional-derivative controller specifically crafted for guiding autonomous nonholonomic differential drive wheeled mobile robots along desired trajectories.. This controller not only showed its performance superiority compared to conventional fuzzy proportional integral derivative controller but also held promise for applications in obstacle detection and collision avoidance. Moving on to [13], a novel method was suggested to achieve accurate trajectory tracking for nonholonomicc mobile robots. This approach combined nonlinear model predictive control, with adjustments to the robot model, objective function, and optimizer to swiftly reduce the steady-state error. The method was rigorously validated through simulations and real plant experiments. In [14], researchers introduced an innovative MPC approach for a nonlinear differential-drive mobile robot, applying input output linearization to design the MPC. The optimization process involved minimizing a quadratic criterion and tuning gains using torques and settling time graphs, with simulation results that highlight its effectiveness.

The authors of [15] presented a control approach that combined predictive and adaptive control techniques for tracking the trajectory of mobile robots. Neural networks and disturbance rejection methods were integrated, and the approach was validated experimentally. In [16], a Mamdani fuzzy controller was introduced for trajectory tracking of nonholonomic mobile robots. This controller demonstrated its ability to accommodate both forward and backward movements, and its advantages were highlighted through simulations, particularly in comparison with forward-only controllers. In [17] researchers introduced a trajectory tracking strategy for nonholonomic wheeled mobile robots, incorporating an adaptive fuzzy observer to handle parameter uncertainties, external disturbances, and a lack of velocity measurements. It effectively combined kinematic and dynamic models to design a control strategy that ensured bounded signals and rapid convergence of the tracking errors, with validation through simulations. Shifting to 2019, [18] investigated the application of MPC in guiding fixed-camera differential wheeled mobile robots. This study focused on achieving trajectory tracking within the camera's field of view and demonstrated enhanced performance through an extended prediction horizon. In [19], a novel controller based on backstepping fuzzy sliding mode was introduced for accurate trajectory tracking of differential wheeled mobile robots. This approach showed improved performance through a combination of simulations and real-world experiments, particularly with an onboard camera-equipped robot.

The authors of [20] introduced a novel kinematic controller specifically designed to track trajectories in autonomous differential drive mobile robots. This controller leveraged an extended nonlinear kinematic model to ensure stability based on Lyapunov theory and enhanced control quality using fuzzy techniques. The feasibility of this approach was validated through computer simulations. Moving on to [21], the introduction of the fuzzy logic controller relied on Z-number theory to establish framework for robust and smooth trajectory tracking in differential wheeled mobile robots. The study demonstrated its efficiency through experiments and a comparative study. In [22], a novel MPC approach is introduced for tracking reference trajectories in constrained linear-time invariant systems. The proposed methodology not only reduces the computational burden, by minimizing the problem, but also demonstrates exceptional efficiency. The material [23] focused on an impressive demonstration of a rapid nonlinear model-based predictive controller, designed for differential drive mobile robots. The controller was implemented with the CasADi toolkit, to achieve both stabilization and accurate trajectory tracking. The study demonstrated the controller’s stability through the extension of optimization horizons, showing, thus, its effectiveness in terminal stabilization and obstacles avoidance.

In [24], researchers present a significant contribution in the form of a controller employing fuzzy adaptive sliding mode designed for wheeled mobile robots derived electrically. This controller not only attains precise trajectory tracking but also exhibits commendable stability, robustness against uncertainties, smooth control, and optimal convergence rate. These attributes are vividly demonstrated through a series of simulations, underlining its potential for real-world application. The researchers in [25] conducted a thorough investigation of trajectory tracking for a two-wheeled mobile robot, comparing the performance of PID and Fuzzy Logic controllers. Indeed, the Fuzzy Logic controller outperforms PID, substantially improving trajectory tracking accuracy by an impressive 50%, a testament to its effectiveness in minimizing trajectory tracking errors.

The aforementioned papers proposed, certainly, solutions for mobile robot tracking problems; however, they consider only the case of the existence of paths of one nature rather than the existence of paths of two or more different natures. To tackle this problem and alleviate the drawbacks of the previous approaches, we propose, in this paper, enhancements to predictive control using fuzzy logic systems. Our main mission is to overcome the instability and tracking accuracy induced by the proposed paths variability. Indeed, based on previous works, this paper addresses the problem of tracking the path of a mobile robot in two different paths in succession. By combining predictive control and fuzzy logic control, a fuzzy predictive control strategy will be designed. The motor error is modelled by a Takagi-Sugeno system, and a predictive controller will be synthesized for the TS fuzzy system, to give the MPC the property of tracking two various paths by the dynamic adjustment of the controller weighting matrices.

The rest of our paper is organized as follows: Section 2 outlines the kinematic model of the mobile robot and its error model. In Section 3, we delve into the synthesis of the proposed fuzzy predictive control technique. Section 4 presents the discussion of simulation findings, and finally, Section 5 concludes the paper.

2 NONHOLONOMIC TWO WHEELED MOBILE ROBOT KINEMATIC MODEL

Figure 1 illustrates the differential drive of the studied mobile robot, a type of mobile robot with a drive system consisting of two wheels positioned on a shared axis. By attaching the local coordinate frame to the robot body, the robot position indicated by $p = [x, y, \theta]$ can also be located in the global (X, Y) reference frame. The differential driving robot, on the other hand, is restricted from sliding sideways by non-holonomic constraints expressed by:

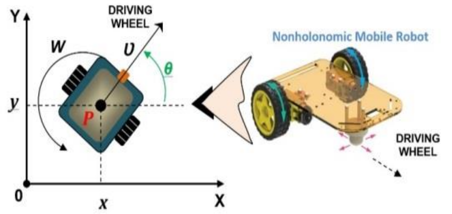
$\dot{y}\cos\theta - \dot{x}\sin\theta = 0$ (1)

A differential drive robot's motion model takes into account both the robot's linear and rotational velocities. When the wheels roll without slipping, the robot's linear velocity consistently aligns with the steering direction, while the consideration of potential spinning movements is factored into the angular velocity. Thus, the mobile robot's kinematic model can be expressed as:

$$\dot{p} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} \quad (2)$$

The kinematic coordinates of the mobile robot are given by the outputs x, y, and θ, while the inputs w and v represent the angular and linear velocities, respectively.

Figure 1. Differential Drive Mobile Robot

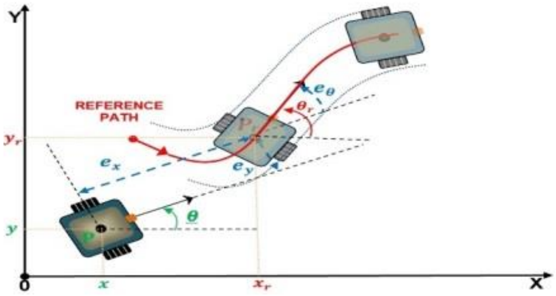


Source: Authors

2.1 KINEMATIC ERROR MODEL

When generating a trajectory for a mobile robot to follow a virtual trajectory with a specified velocity profile, the posture error refers to the disparity between the virtual robot's position and orientation and that of the physical robot, as depicted in Figure 2 [26].

Figure 2. Evolution of Robot Tracking Error Transformation.



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The vector $p_r = [x_r \quad y_r \quad \theta_r]^T$ represents the reference robot's posture, whereas the vector $p = [x \quad y \quad \theta]^T$ represents the physical robot's posture. The distinction in postures is represented by the vector $e = [e_x \quad e_y \quad e_\theta]^T$ and determined as the difference between the reference and physical robot postures, given as:

$$e = \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (3)$$

The mobile robot dynamics are described by Eq. (2) and the deriving Eq. (3) leads to the following kinetic model:

$$\dot{e} = \begin{bmatrix} \cos(e_\theta) & 0 \\ \sin(e_\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ w_r \end{bmatrix} + \begin{bmatrix} -1 & e_y \\ 0 & -e_x \\ 0 & -1 \end{bmatrix} u \quad (4)$$

While v_r and w_r represent virtual robot velocities and act as linear and angular feed-forward control inputs, respectively. They may be stated as:

$$\begin{cases} v_r = \pm\sqrt{\dot{x}_r^2 + \dot{y}_r^2} \\ w_r = (\dot{x}_r^2 \ddot{y}_r^2 - \dot{y}_r^2 \ddot{x}_r^2)/(\dot{x}_r^2 + \dot{y}_r^2) \end{cases} \quad (5)$$

u designates the control input obtained by combining the input for feed forward and feedback control is as follows:

$$u = u_f + u_p = \begin{bmatrix} v_r \cos(e_\theta) \\ w_r \end{bmatrix} + \begin{bmatrix} v \\ w \end{bmatrix} \quad (6)$$

u_f denotes the feedforward control input derived through a nonlinear transformation of the reference inputs, whereas u_p denotes the resultant output from the suggested controller.

The tracking-error model is a result of (6) and (4), i.e.:

$$\dot{e} = \begin{bmatrix} 0 & w & 0 \\ -w & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} e + \begin{bmatrix} 0 \\ \sin(e_\theta) \\ 0 \end{bmatrix} v_r + \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} u_p \quad (7)$$

After linearizing the error model (7) around the reference trajectory (with $e_x = e_y = e_\theta = 0$, $u_p = 0$), the following linear kinematic error model is derived:

$$\dot{e} = \begin{bmatrix} 0 & w_r & 0 \\ -w_r & 0 & v_r \\ 0 & 0 & 0 \end{bmatrix} e + \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} u_p \quad (8)$$

Equation (8) is the state space representation. The assurance of its controllability remains valid as long as either v_r or w_r stays non-zero. As a result, the robust control procedure should be developed to accurately guide the physical robot and ensure it precisely matches the reference target trajectory.]

3 FUZZY PREDICTIVE CONTROL

3.1 T-S FUZZY MODEL

The T-S fuzzy model serves to be a valuable instrument for both modelling and controlling dynamical systems. It has the capacity to mimic complex systems with acceptable precision while using fewer rules, endowing it with significant engineering expertise. Furthermore, the T-S fuzzy modell used for linear systems enables the development of localized linear controllers and provides an accurate description of the system dynamics [27]. In this context, the dynamic association may be defined as a linear and confined link constructed within the framework of the state space representation. Using various values within the v_r and w_r ranges leads to the formation of several linear error models. These local models are then established to build the T-S fuzzy model [28].

The T-S fuzzy systems could create dynamic systems in discrete-time state space that change over time. Indeed, let us deal with the following T-S fuzzy logic system composed of R rules of the form:

$R_i : if\ v_r(k)\ is\ P_1^K\ and\ w_r(k)\ is\ P_2^L\ then\ e^i(k+1) = A_i e(k) + b_i u(k)$ (9)

v_r and w_r serve as premise variables, representing the virtual robot velocities that change when the discrete-time parameter k changes ($k \in \{0,1,2,3,\ldots.\}$). The fuzzy sets are represented byP_1^K,P_2^L, and $i = 1,2,3,\ldots R$ with R represented the number of fuzzy rules. A_i, b_i represent the state and input matrices, respectively.

In this context, the state-space difference Equation may be expressed as:

$e(k+1) = (\sum_{i=1}^{R} \mu_i(k)(A_i e(k) + b_i u(k)))/(\sum_{i=1}^{R} \mu_i(k))$ (10)

Assuming $\zeta_i(k) = \mu_i(k)/\sum_{i=1}^{R} \mu_i(k)$ the fuzzy basis functions for the i^{th} fuzzy rule and $(k) = \zeta_1 A_1 + \zeta_2 A_2 + \cdots + \zeta_R A_R$, and$b(k) = \zeta_1 b_1 + \zeta_2 b_2 + \cdots + \zeta_R b_R$, the expression (10) can be reformulated as follows:

$e(k+1) = A(k)e(k) + b(k)u(k)$ (11)

As a result, we have a model equivalent to the previous one and describes the dynamic evolution of $e(k)$. The matrices $A(k)$ and $b(k)$ change as the fuzzy basis functions do, and all of these changes are influenced by time.

3.2 CONTROL STRATEGY

The predictive control idea is commonly employed in improving the performance of trajectory tracking. It is being researched to improve robot control by using a receding-horizon technique to determine control inputs over a specific time frame N_p . The goal is to decrease the tracking error between desired and projected trajectories, which can be performed using the following quadratic cost function:

$J(u_p, k) = \sum_{i=1}^{N_p} \left(e_r(k+i) - e(k+i)\right)^T Q_f \left(e_r(k+i) - e(k+i)\right) + u_p^T(k,i) R_f u_p(k,i)$ (12)

Through the minimization of this cost function (12), the control variable values can be tuned to guide the robot in the direction of the desired trajectory, where $\mathrm{e_r}$and e represent the reference trajectory that the robot should follow and the tracking error, respectively. The weighting matrices $\mathrm{Q_f}$ and $\mathrm{R_f}$ can be dynamically changed using a fuzzy system as shown in Figure 3, with $\mathrm{Q_f} \in \mathfrak{R}^{n} \times \mathfrak{R}^{n}$ and $\mathrm{R_f} \in \mathfrak{R}^{m} \times \mathfrak{R}^{m}$ and $\mathrm{Q_f} \geq 0$ and $\mathrm{R_f} \geq 0$.

With the shifting time frame in mind, the model's output projection, at time instance$\mathrm{N_p}$, may be expressed as:

$$\mathrm{e}(\mathrm{k}+\mathrm{N_p}) = \prod_{\mathrm{j}=1}^{\mathrm{N_p}-1} \mathrm{A}(\mathrm{k}+\mathrm{j})\mathrm{e}(\mathrm{k}) + \sum_{\mathrm{i}=1}^{\mathrm{N_p}} \left(\prod_{\mathrm{j}=1}^{\mathrm{N_p}-1} \mathrm{A}(\mathrm{k}+\mathrm{j}) \right)$$

$$\times \mathrm{B}(\mathrm{k}+\mathrm{i}-1)\mathrm{u_p}(\mathrm{k}+\mathrm{i}-1) + B(k+N_p-1)u_p(k+N_p-1) \quad (13)$$

The following is a description of the prediction-error vector, which offers information on how precisely the robot is following the specified trajectory:

$$E_p(k) = \begin{bmatrix} e(k+1)^T & e(k+2)^T & \dots & e(k+N_p)^T \end{bmatrix}^T \quad (14)$$

In addition,

$$U_p(k) = \begin{bmatrix} u_p^T(k+1)^T & u_p^T(k+2)^T & \dots & u_p^T(k+N_p-1)^T \end{bmatrix}^T \quad (15)$$

If we consider that,

$$G(k) = [A(k) \quad A^2(k) \quad \dots \quad A^{N_p}(k)]^T \quad (16)$$

And

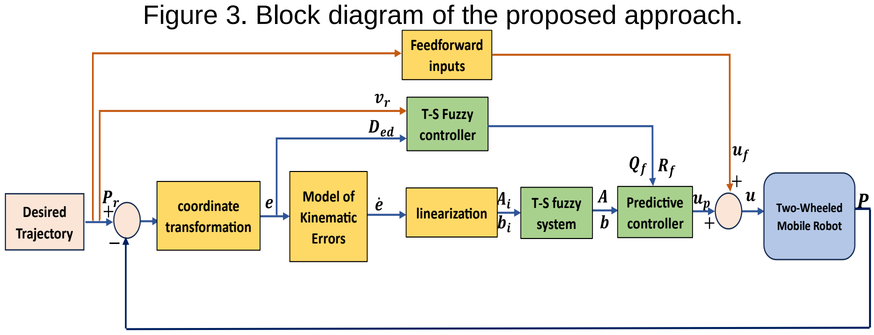
$$H(k) = \begin{bmatrix} B(k) & 0 & \dots & 0 \\ A(k)B(k) & B(k) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N_p}(k)B(k) & A^{N_p-1}(k)B(k) & \dots & B(k) \end{bmatrix} \quad (17)$$

The robot tracking prediction-error vector becomes:

$$E_p(k) = G(k)e(k) + H(k)U_p(k) \quad (18)$$

With $G \in \Re^{n.N_p} \times \Re^n$ and$H \in \Re^{n.N_p} \times \Re^{m.N_p}$.

Figure 3. Block diagram of the proposed approach.



Source: Authors

Access to the next point of the reference trajectory, that the robot should follow, is required for successful trajectory tracking. It would be difficult to create a control rule that directs the robot to accurately track this future reference point without knowledge of it. This implies that future control errors should decrease based on the dynamics given by the reference model matrix (A_r). To do this, we will need to choose a reference error-tracking trajectory, which can be described in state space for $i = 1, \ldots, N_p$ as follows:

$$e_r(k+i) = A_r^i e(k) \quad (19)$$

Let us suppose that the vector formulation of the robot reference, that specifies the tracking error, is represented as:

$$E_p^r = \begin{bmatrix} e_r(k+1)^T & e_r(k+2)^T & \ldots & e_r(k+N_p)^T \end{bmatrix}^T \quad (20)$$

Through the use of Eqs. (19) and (20), the reference tracking error vector of the robot is derived in the following form:

$$E_p^r(k) = G_r e(k) \quad (21)$$

Where

$$G_r = \begin{bmatrix} A_r & A_r^2 & \ldots & A_r^{N_p} \end{bmatrix}^T \quad (22)$$

with $E_p^r \in \Re^{n.N_p}$ over the whole observation period N_p , and $G_r \in \Re^{n.N_p} \times \Re^n$.

The cost function is established to choose control inputs in the model predictive control frame and (12) can be re-written as follows:

$$J(U_p) = \left(E_p^r - E_p\right)^T \widetilde{Q_f} \left(E_p^r - E_p\right) + U_p^T \widetilde{R_f} U_p \quad (23)$$

Obtaining the adequate control law means minimizing the cost function in the following way:

$$\frac{\partial J}{\partial U_p} = -2\widetilde{Q}_f H^T E_p^r + 2H^T \widetilde{Q}_f E_p + 2\widetilde{R}_f U_p \quad (24)$$

And thus, the control vector can be calculated using Eqs. (18), (21), and (24) as follows:

$$U_p(k) = \left(H^T \widetilde{Q}_f H + \widetilde{R_f}\right)^{-1} H^T \widetilde{Q}_f (G_r - G) e(k) \quad (25)$$

Where Q_f and R_f are the weighting matrices utilized to create the objective function in the MPC optimization problem.

In the suggested methodology, we aim to enhance the responsiveness of the MPC controller to variations in the system and elevate the accuracy of trajectory tracking. This improvement is achieved through the dynamic adjustment of weighting matrices, allowing the controller to adapt more effectively to changes in the system dynamics and optimize its performance in tracking desired trajectories. This adjustment is done by a fuzzy logic system that takes into consideration two essential inputs: the virtual robot's linear velocity v_r and the current distance D_{ed} between the mobile robot and the virtual moving robot as shown in Figure 5 with $Q_f = diag(q_1, q_2, q_3)$, $R_f = diag(r_1, r_2)$.

$$\widetilde{Q_f} = \begin{bmatrix} Q_f & 0 & \cdots & 0 \\ 0 & Q_f & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q_f \end{bmatrix} \text{ and } \widetilde{R_f} = \begin{bmatrix} R_f & 0 & \cdots & 0 \\ 0 & R_f & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_f \end{bmatrix}$$

This indicates that $\widetilde{Q_f} \in \Re^{n.N_p} \times \Re^{n.N_p}$ and $\widetilde{R_f} \in \Re^{m.N_p} \times \Re^{m.N_p}$

Using (25), we can now describe the feedback controlllaw for model predictive control with the following expression:

$u_p(k) = K_{mpc}.e(k)$ (26)

And K_{mpc} is defined as the first m rows of the matrix $\left[\left(H^T\widetilde{Q_f}H + \widetilde{R_f}\right)^{-1}H^T\widetilde{Q_f}(G_r - G)\right]$ with $K_{mpc} \in \Re^m \times \Re^n$. Figure 3. depicts the mechanism of the proposed controller.

4 SIMULATION RESULTS

To assess the effectiveness of the suggested control procedure, a series of simulations were executed in MATLAB. This segment specifically focused on examining the mobile robot's performance as it navigated through two distinct trajectories, providing a comprehensive analysis of the strategy's applicability and performance under varied conditions.

Both simulation examples have constraints on velocity and acceleration as follows: $u_1 \leq 1.5\ m/s$ and$-10 \leq u_2 \leq 10\ rad/s$, $v_r(t)$ ranges of $[0; 1.5]\,m/s$ and $w_r(t)$ ranges of $[-10, 10]\ rad/s$, prediction horizon$N_p = 4$. Utilizing the lower and upper limits of reference velocities, we identified the subsequent subsystems:

$$A_1 = \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, b_1 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 10 & 0 \\ -10 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, b_2 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & 1.5 \\ 0 & 0 & 0 \end{bmatrix}, b_3 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 10 & 0 \\ -10 & 0 & 1.5 \\ 0 & 0 & 0 \end{bmatrix}, b_4 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}$$

The used membership functions in the T-S fuzzy model are displayed in Figure 4. The T-S fuzzy rules are disposed as follows:

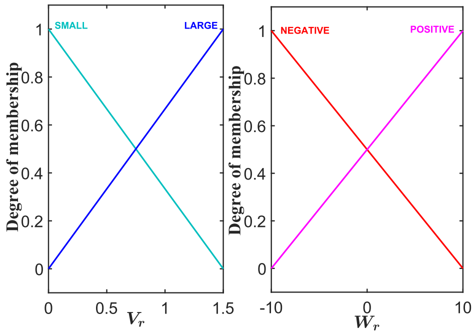
R_1: if $v_r(k)$ is small and $w_r(k)$ is negative then $e^1(k+1) = A_1e(k) + b_1u(k)$

R_2 : if $v_r(k)$ is small and $w_r(k)$ is positive then $e^2(k+1) = A_2e(k) + b_2u(k)$

R_3 : if $v_r(k)$ is large and $w_r(k)$ is negative then $e^3(k+1) = A_3e(k) + b_3u(k)$

R_4 : if $v_r(k)$ is large and $w_r(k)$ is positive then $e^4(k+1) = A_4 e(k) + b_4 u(k)$

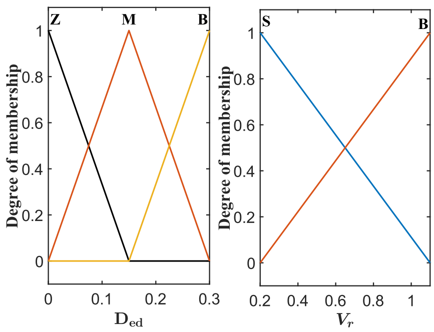
Figure 4. Membership functions of the T-S fuzzy model



Source: Authors

In both simulation scenarios, the weighting matrices for the suggested control rule are carefully determined through the application of a T-S fuzzy logic system. This specialized system takes into account crucial inputs such as the current distance D_{ed} between the mobile and virtual robots, as well as the linear velocity v_r. The outputs, denoted as $(q_1, q_2, q_3, r_1, r_2)$, play a pivotal role in the control strategy. The parameter D_{ed} is systematically categorized into three distinct fuzzy sets, ranging from Zero to Big, with the transition passing through Medium. This categorization effectively covers the entire range from 0 to 0.3 meters. Meanwhile, the velocity v_r is characterized by a range of $[0.2; 1.1]\, m/s$, classified into two fuzzy sets: Small and Big. These configurations are thought fully illustrated in Figure 5. The precise fuzzy rules that dictate the determination of both Q_f and R_f values are summarized in Tables (1) and (2).

Figure 5. Membership functions for the T-S fuzzy system inputs.



Source: Authors

Table 1. Fuzzy rules for Q and R.

Rules	D_{ed}	v_r	q_1	q_2	q_3	r_1	r_2
R_1	*Zero*	*Small*	*low*	*low*	*Medium*	*low*	*low*
R_2	*Zero*	*Big*	*very low*	*low*	*high*	*low*	*low*
R_3	*Medium*	*Small*	*Medium*	*Medium*	*low*	*Medium*	*Medium*
R_4	*Medium*	*Big*	*Medium*	*Medium*	*low*	*Medium*	*Medium*
R_5	*Big*	*Small*	*high*	*high*	*very low*	*high*	*high*
R_6	*Big*	*Big*	*high*	*high*	*very low*	*high*	*high*

Source: Authors

Table 2: Values of linguistic outputs.

Linguistic outputs	q_1	q_2	q_3	r_1	r_2
Very low	*1*	*/*	*0.01*	*/*	*/*
Low	*2*	*80*	*0.03*	*0.0001*	*0.0001*
Medium	*5*	*90*	*0.05*	*0.001*	*0.001*
High	*10*	*100*	*2*	*0.01*	*0.01*

Source: Authors

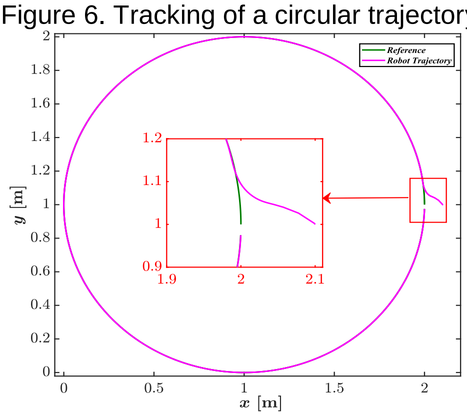
4.1 TRACKING A CIRCULAR TRAJECTORY

In this scenario, a mobile robot is assigned the task of tracking the circular trajectory given by (27).

$$\begin{cases} x_r(t) = 1.0 + \cos\left(\frac{2\pi t}{30}\right) \\ y_r(t) = 1.0 + \sin\left(\frac{2\pi t}{30}\right) \end{cases} \quad (27)$$

The reference robot maintains a consistent linearrvelocity $\mathrm{v_r} = 0.2\ \mathrm{m/s}$ and angular velocity$\mathrm{w_r} = 0.2\ \mathrm{rad/s}$. The initial positionnof the robot is set at $\mathrm{p}(0) = [2.1 \quad 1.0 \quad 2.0]$.

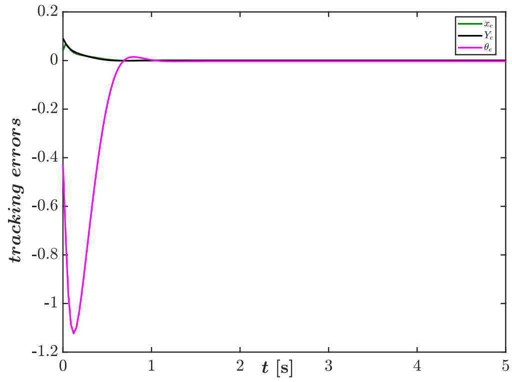
Figure 6. Tracking of a circular trajectory.



Source: Authors

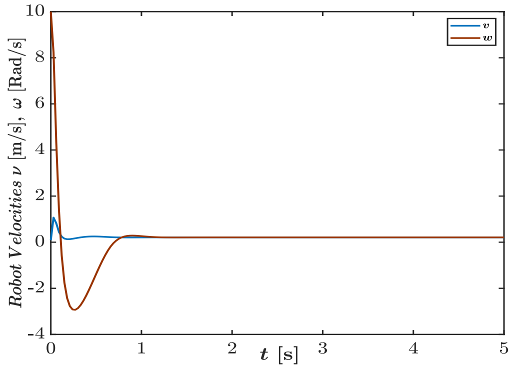
Figure 6 offers a visual representation of the robot's trajectory under the proposed fuzzy predictive control law and demonstrates the robot's successful adherence to the desired path.

Figure 7. Tracking errors.



Source: Authors

Figure 8. Robot’s linear and angular speeds.



Source: Authors

Additionally, Figure 7 and 8 show the tracking errors and control inputs of the robot, respectively.

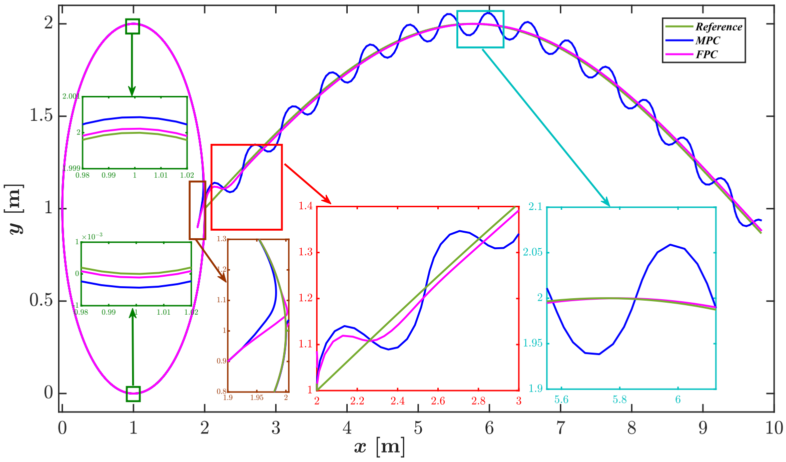
4.2 TRACKING CIRCULAR AND SINUSOIDAL TRAJECTORIES

In this part, we compare the results of the proposed Fuzzy Predictive Control (FPC) to those of the Model Predictive Control (MPC) established in [29] The primary goal is to direct the mobile robot along a dynamic course with varying linear and rotational velocities. This starts with a circular path given by (27), then a sinusoidal path defined by (28). The robot's initial location is $p(0) = [1.9 \quad 0.9 \quad \pi/3]$.

$$\begin{cases} x_r(t) = 2.0 + t \\ y_r(t) = 1.0 + \sin\left(\frac{2\pi t}{30}\right) \end{cases} \quad (28)$$

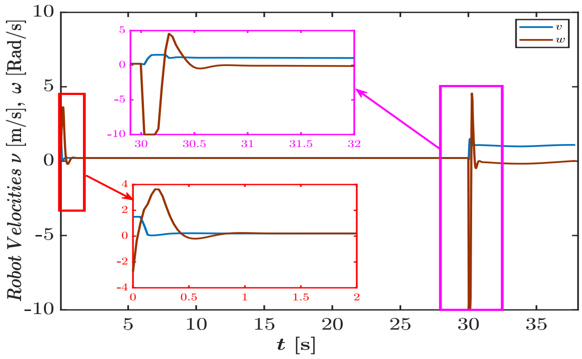
Figure 9 depicts an illustration of the mobile robot's trajectory under (FPC) and (MPC) control techniques. Additionally, Figure 10 shows the control inputs used in the proposed control technique FPC, whereas Figure 11 illustrates the control inputs used in the controller (MPC).

Figure 9. Tracking circular and sinusoidal paths.



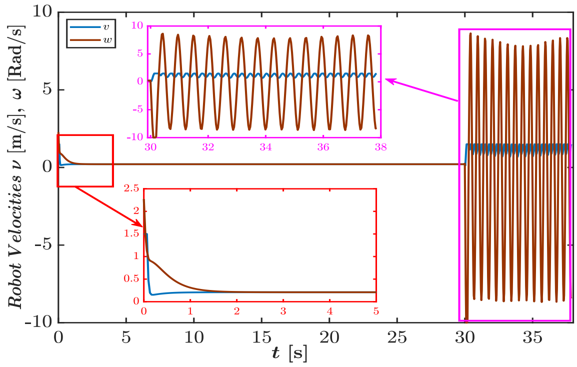
Source: Authors

Figure 10. Linear and angular speeds using FPC approach.



Source: Authors

Figure 11. Linear and angular speeds using MPC approach.



Source: Authors

Furthermore, we investigate tracking errors using the Integral Square Error (ISE), with the results presented in Table (3). The results in Table (3) highlight the effectiveness and the performance superiority of the proposed fuzzy predictive controller (FPC) in coordinating mobile robot movements along trajectories with variable linear and rotational velocities.

Table 3: Tracking errors analysis.

	$ISE(e_x)$	$ISE(e_y)$	$ISE(e_\theta)$
FPC	*0.0011*	*0.0015*	*0.2392*
MPC	*0.0028*	*0.0159*	*2.263*

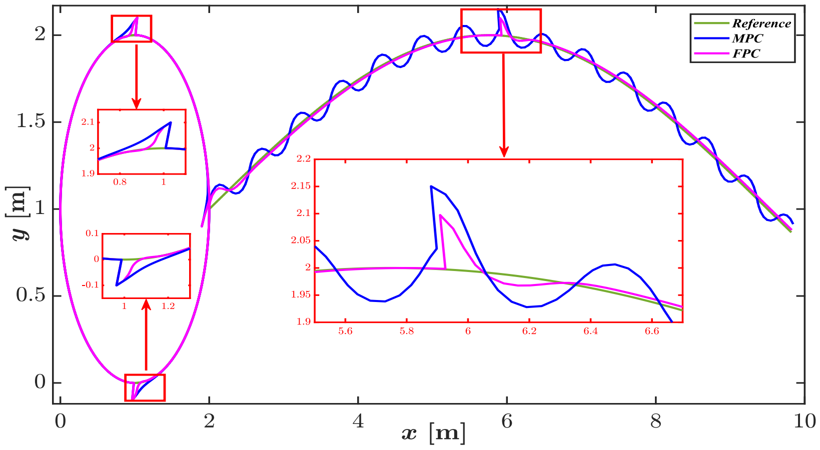
Source: Authors

4.3 ROBUSTNESS TEST

Conducting robustness testing is necessary to ensure stability and achieve optimal performance of control systems in the presence of external disturbances. These tests hold valuable importance in achieving robustness within control systems.

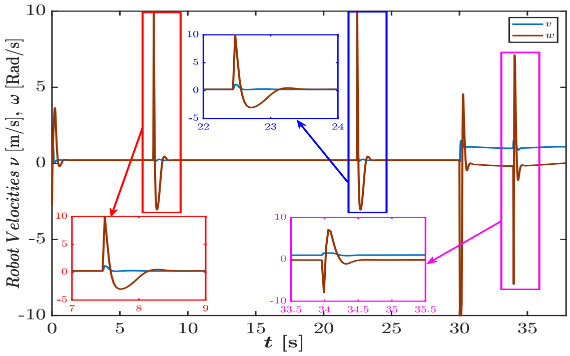
In our case, we are looking at how external disturbances impact the position of the mobile robot at the three points illustrated in Figure 12.

Figure 12. Robustness of tracking circular and sinusoidal paths.



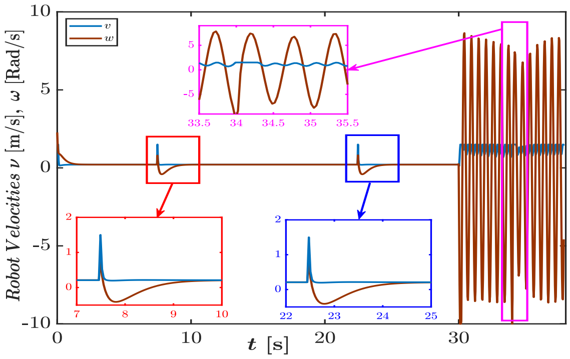
Source: Authors

Figure 13. Illustrates the robustness test of angular and linear speeds using the FPC approach



Source: Authors

Figure 14. Illustrates the robustness test of angular and linear speeds using the MPC approach



Source: Authors

When perturbations are introduced, our suggested solution successfully and quickly directs the robot back to the reference path. Notably, when perturbations are used, the proposed approach (FPC) beats the method (MPC) in perturbation rejection. Figure 13 and 14 show the turbulent velocities of the robot, demonstrating the quick dynamics in returning to the reference path. This validates the system's increased robustness when using our technique. Based on these results, we conclude that our solution (FPC) maintains system stability, achieves high tracking accuracy, ensures faster dynamics, and outperforms (MPC) in terms of overall performance.

5 CONCLUSION

The deal of the paper addressed the task pertaining to the navigation of mobile robots through trajectory tracking. The proposed control strategy not only successfully manages the tracking of two distinct paths but also ensures stability by accommodating trajectory changes within the reference path. This is achieved by employing a T-S fuzzy system and integrating predictive control technique. The real-time adjustment of the control weighting matrices is facilitated by a fuzzy system. The outcomes of the simulation confirmed the robustness of the suggested approach, showed its tracking performance, the swift convergence of the tracking errors toward zero signifies the gradual approach of the tracking errors to zero, and the ensured smoothness of tracking operation. These results underscore the potential and effectiveness of the suggested control methodology in effectively resolving the complex trajectory tracking challenge, particularly in scenarios involving varying paths.

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