Studies in Engineering and Exact Sciences
ISSN: 2764-0981

Multi-objective optimization of machining conditions by geometric programming

Otimização multiobjetivo das condições de usinagem por programação geométrica

DOI: 10.54021/seesv5n1-022

Recebimento dos originais: 16/01/2024
Aceitação para publicação: 23/02/2024

Mohamed Djennane
PhD in Mechanical Engineering
Institution: Department of Mechanical Engineering, National Polytechnic School Constantine
Address: Constantine, 25000, Algeria
E-mail: moh_djen@yahoo.fr

Rachid Benbouta
PhD in Mechanical Engineering
Institution: Laboratory LIECS_MS, Department of Mechanical Engineering, University of BATNA 2
Address: Batna, 5000, Algeria
E-mail: r_benbouta@yahoo.fr

Allaoua Kherraf
PhD in Mechanical Engineering
Institution: Laboratory LRP, Department of Mechanical Engineering, University of BATNA 2
Address: Batna, 5000, Algeria
E-mail: all_kherraf@outlook.fr

ABSTRACT

In metal cutting processes, cutting conditions have an influence on reducing the production cost and time and deciding the quality of a final product. This paper outlines the development of an optimization strategy to determine the optimum cutting parameters for turning processes. Two objective functions are simultaneously optimized under a set of practical of machining constraints, the first objective function is production cost and the second one is the production time. The optimal values of the cutting conditions are found based on the objective function developed for the typified criterion by using a non-linear programming technique called “geometric programming”. In the optimization procedure, the objective functions are subject to constraints of maximum and minimum feed rates and speeds available, cutting power, tool life, deflection of work piece, axial pre-load and surface roughness. An example is presented to illustrate the procedure of this technique.

Keywords: cutting parameters, geometric programming, multi-criteria optimization, turning processes.

RESUMO
Nos processos de corte de metal, as condições de corte têm influência na redução do custo e do tempo de produção e na decisão da qualidade de um produto final. Este artigo descreve o desenvolvimento de uma estratégia de otimização para determinar os parâmetros de corte ideais para processos de torneamento. Duas funções objetivas são simultaneamente otimizadas sob um conjunto de restrições práticas de usinagem, a primeira função objetiva é o custo de produção e a segunda é o tempo de produção. Os valores ótimos das condições de corte são encontrados com base na função objetiva desenvolvida para o critério tipificado usando uma técnica de programação não linear chamada "programação geométrica". No procedimento de otimização, as funções objetivas estão sujeitas a limitações de velocidades e velocidades máximas e mínimas de alimentação, potência de corte, vida útil da ferramenta, deflexão da peça de trabalho, pré-carga axial e rugosidade da superfície. Um exemplo é apresentado para ilustrar o procedimento dessa técnica.

Palavras-chave: corte de parâmetros, programação geométrica, otimização de múltiplos critérios, processos de torneamento.

1 INTRODUCTION

Every machining process requires careful consideration when choosing the best cutting parameters, such as the number of passes, depth of cut for each pass, feed rate, and cutting speed [1].

A cutting procedure in turning operations can be done in a single pass or numerous passes. Multi-pass turning is preferable over single pass turning in the industry for economic reasons [2]. When there are numerous practical restrictions to consider, the optimization problem of machining parameters in multi-pass turnings becomes extremely difficult [3].

Multiobjective optimization, i.e., the optimization of multiple objectives at the same time, has been an active area of research since the early works of Edgeworth and Pareto. Edgeworth introduced an optimality notion for such problems in his book in 1881 [4] and Pareto in his book on political economy in 1906 [5]. For a short historical overview see Eichfelder and Jahn [6].

The literature in the area of the optimization of multi-pass machining operations has therefore been quite limited. Crookall and Venkataramani [7], Kals *et al*., [8] and Lambert and Walvekar [9] have analyzed the optimization of multi-pass turning operations. However, because they all chose the number of passes at random, their results cannot be regarded as the optimal values.

A special kind of nonlinear programming known as a Geometric Programming Problem (GPP) is frequently utilized in applications for production planning, personnel allocation, distribution, risk management, chemical process designs, and other engineer design problems.

The Geometric Programming Problem is a unique method for determining the best values for posynomial and signomial functions. In the classical optimization technique, a system of nonlinear equations is generally faced after taking partial derivatives for each variable and equalizing them to zero. Since the objective function and the constraints in the GPPs will be in posynomial or signomial structures, the solution of the system of nonlinear equations obtained by the classic optimization technique will be very difficult. The solution to the GPP follows the opposite method with respect to the classical optimization technique and it depends on the technique of first finding the weight values and calculating the optimum value for the objective function, then finding the values of the decision variables [10,11].

Numerous multi-objective programming issues are solved via numerical approximations. The Taylor series expansion, which is also provided as a solution method in this paper, is one of the numerical approximations. Taylor series were employed by Toksar [12] and Güzel and Sivri [13] to address the multi-objective linear fractional programming issue and provided examples.

In this study, the algorithm used is adopted from the study of Agapiou [14, 15], Wang [16], Djennane *et al*. [17], Sofuoglu *et al*. [18] and Djari *et al*. [19] which are proposed for turning operations. The optimization method proposed consists in the formulation of the objective function based on two economic criteria, the production time and the cost of production. However, the methodology used in this work is based on the geometric programming to which we have introduced modifications and developed a program for that purpose. This numerical approach minimizes the weighted objective function subject to Kuhn-Tucker Conditions expanded the first order Taylor series expansion about any arbitrary initial feasible solution.

In this research, we present a weighting-based approach to the problem of finding the compromise optimal solution of certain multi-objective geometric programming problems when the cost coefficients are continuous functions. First of all, the multiple objective functions transformed to a single objective by considering it as the linear combination of the multiple objectives along with suitable constants called weights.

2 FORMULATION OF THE MULTI-CRITERIA OBJECTIVE FUNCTION

Machining parameter optimization models are mathematical models formulated for realistic machining processes. These models have objective functions based on certain economic criteria and subject to various practical constrains from machining conditions and quality specifications. The formulation of process models requires the knowledge of mathematical equations to represent die relations of economical and physical parameters for the machining process and the knowledge on die whole machine-tool-workpiece system.

2.1 OPTIMIZATION CRITERIA

In die optimization of machining parameters, objective functions are mathematical formulations governed by certain production criteria. They are the basis on which machining parameters are optimized.

The advantage of optimizing the cutting conditions lies in the improvement of certain economic and technical judgment criteria of machining that the production cost, the productivity, the service life of cutting tools and in rare studies the surface condition produced [16, 20-23].

2.1.1 Production Time

The total time required to produce a part is the sum of the time required for machining, tool change, tool quick return, and assembly and dismounting time [14, 15, 24-26].

$$T_u = t_m + t_{cs}\left(\frac{t_m}{T}\right) + t_R + t_h \tag{1}$$

$$t_m = \frac{\pi.D.L}{100.V.f} \tag{2}$$

From the Taylor law developed later by Gilbert, the duration of the tool life *T* is given by the following equation:

$$T = K^{\frac{1}{a_3}}.V^{\frac{1}{a_3}}.f^{\frac{a_1}{a_3}}.d^{\frac{a_2}{a_3}} \tag{3}$$

By replacing t_m and *T* by their respective expressions (2) and (3) in equation (1), we obtain the time production in the following form:

$$T_u = A.V^{-1}.f^{-1} + A.V^{\frac{1-a_3}{a_3}}.f^{\frac{a_1-a_3}{a_3}}.d_c^{\frac{a_2}{a_3}}.K^{\frac{-1}{a_3}}.t_{cs} + t_h + t_R \tag{4}$$

Where:

$$A = \frac{\pi.D.L}{1000}$$

2.1.2 Production Cost

If material cost is not considered, unit production cost C_u ($/piece) en be expressed by Armarego [27].

$$C_u = C_m + C_i + C_r + C_t \tag{5}$$

Where C_m ($/piece), C_i ($/piece), C_r ($/piece) and C_t ($/piece) are actual machining cost, machine idle cost, tool replacement cost and tool cost, respectively. This expression in equation (5) has been widely accepted by many researchers in this field [28].

Each cost term in equation (5) is analyzed as follows:

$$C_m = k_0\left[\frac{\pi DL}{1000V_r f_r}\left(\frac{d_c - d_f}{d_r}\right) + \frac{\pi DL}{1000V_f f_f}\right] \tag{6}$$

$$C_i = k_0\left[t_c + \left(h_1 L + h_2\right)\left(\frac{d_c - d_f}{d_r} + 1\right)\right] \tag{7}$$

$$C_r = k_0 \frac{t_c}{T_p}\left[\frac{\pi DL}{1000 V_r f_r}\left(\frac{d_c - d_f}{d_r}\right) + \frac{\pi DL}{1000 V_f f_f}\right] \tag{8}$$

$$C_t = \frac{k_t}{T_p}\left[\frac{\pi DL}{1000 V_r f_r}\left(\frac{d_c - d_f}{d_r}\right) + \frac{\pi DL}{1000 V_f f_f}\right] \tag{9}$$

2.2 THE MACHINING CONSTRAINTS

Optimal cutting conditions should satisfy some technological constraints. Machine tool, cutting tool and workpiece specifications are the sources of these restrictions. These constraints can be determined experimentally for a given workpiece as a function of tool material and geometry, etc. Otherwise, the constraints related to speed, feed and depth of cut for the particular tool workpiece combination should be used in order to proceed with optimization of cutting parameters. In this work, we consider the following limitations:

- Cutting speed: $V_{min} \le V \le V_{max}$ (10)

- Feed rate: $f_{min} \le f \le f_{max}$ (11)

- Depth of cut: $d_{min} \le d \le d_{max}$ (12)

- The maximum power available for the turning operation will be a constraint to be taken into account. This power is given by Agapiou [14, 15]:

$$0.0373.V^{0.91}.f^{0.78}.d^{0.75} \le HP_{max} \tag{13}$$

- In the turning operations, the obtained surface roughness must be smaller than the specified value, SR_{max} , given by technological criteria, so that the following equation is satisfied [14, 15]:

$$14.785.V^{-1.25}.f^{1.004}.d^{0.25} \leq SR_{max} \quad (14)$$

- The temperature constraint is given by the inequality:

$$74.96.V^{0.4}.f^{0.2}.d^{0.105} - 17.8 \leq \theta_{max} \quad (15)$$

- The maximum amount of cutting forces F_{max} should not exceed a certain value as higher forces produce shakes and vibration. This constraint is given below.

$$844.V^{-0.1013}.f^{0.725}.d^{0.75} \leq F_{max} \quad (16)$$

2.3 THE WEIGHTING METHOD TO THE MULTI-OBJECTIVE GEOMETRIC PROGRAMMING PROBLEM

Optimization with multiple objectives is currently a very active field of research. The method used consists in defining an objective function by combining the time criterion and production cost in the same function.

Optimization with multiple objectives is currently a very active field of research. The method used consists in defining an objective function by combining the time criterion and production cost in the same function.

The weighting method is the simplest multi-objective optimization which has been widely applied to find the non-inferior optimal solution of multi-objective function within the convex objective space.

If $f_1(x)$, $f_2(x)$,……,$f_n(x)$ are n objective functions for any vector $x = (x_1, x_2 \ldots. x_n)^T$ then we can define weighting method for their optimal solution as defined below [29]:

Let $W = \left\{ w : w \in R^n,\ w_j \succ 0,\ \sum_{j1}^{n} w_j = 1 \right\}$ (17)

to be the set of non-negative weights. The weighted objective function for the multiple objective function defined above can be defined as *P(w)* where:

$$P(w) = \min_{x \in X} \sum_{j=1}^{n} w_j f_j(x) \quad (18)$$

subject to

$$f_i(x) \le 1; \quad i = 1, 2, ..., m \quad (19)$$

$$x_j \succ 0; \quad j = 1, 2, ..., n \quad (20)$$

It must be noted that the optimal solution of a weighting problem should not be used as the best compromise solution, if the weights do not reflect the decision maker's preferences or if the decision maker does not accept the assumption of a linear utility function. For more details about the weighted method refer [30].

One can often classify the objectives according to their importance but the weights will be generally found by trial and error [31]. For better answering the problem, we use the Duffin's technique [32] that the combination of the functions.

Our objective will be summed up in the problem of minimizing an objective function in the form of a weighted sum of time and cost of production, taking into account the limitations (10), (11) and (12), and the constraints related to physics (13), (14), (15) and (16). Using the system (18), the problem will be formulated as follows:

$$\left| \begin{array}{ll} \text{Minimize} & Y = a_1.C_u + a_2.T_u \\ \text{such as} & V_{min} \le V \le V_{max} \\ & f_{min} \le f \le f_{max} \\ & d_{min} \le d \le d_{max} \\ \text{and} & 0.0373.V^{0.91}.f^{0.78}.d^{0.75} \le HP_{max} \\ & 14.785.V^{-1.25}.f^{1.004}.d^{0.25} \le SR_{max} \\ & 74.96.V^{0.4}.f^{0.2}.d^{0.105} - 17.8 \le \theta_{max} \\ & 844.V^{-0.1013}.f^{0.725}.d^{0.75} \le F_{max} \end{array} \right. \quad (21)$$

With

$$a_1 + a_2 = 1 \quad \text{and} \quad a_1,\ a_2 \ge 0$$

By replacing C_u and T_u by the respective expressions (4) and (5) in equation (21) we obtain:

$$Y = a_1.\left[C_o.A.V^{-1}.f^{-1} + A.V^{\frac{1-a_3}{a_3}} f^{\frac{a_1-a_3}{a_3}}.d_c^{\frac{a_2}{a_3}}.K^{\frac{1}{a_3}}.(C_o.t_{cs} + C_t) + C_o(t_h + t_R)\right] +$$
$$a_2.\left[A.V^{-1}.f^{-1} + A.V^{\frac{1-a_3}{a_3}}.f^{\frac{a_1-a_3}{a_3}}.d_c^{\frac{a_2}{a_3}}.K^{\frac{-1}{a_3}}.t_{cs} + t_h + t_R\right]$$

After transformation we obtain the following form:

$$Y = [a_1.C_o.(t_h + t_R) + a_2(t_h + t_R)] + (a_1.C_o.A + a_2.A).V^{-1}.f^{-1}$$
$$+[a_1.A.(C_o.t_{cs} + C_t) + a_2.A.t_{cs}].V^{\frac{1-a_3}{a_3}} f^{\frac{a_1-a_3}{a_3}}.d_c^{\frac{a_2}{a_3}}.K^{\frac{1}{a_3}} \quad (22)$$

The first term of equation (22) is constant. We can write:

$$Y = C_{01}.V^{-1}.f^{-1} + C_{02}.V^{\frac{1-a_3}{a_3}} f^{\frac{a_1-a_3}{a_3}}.d_c^{\frac{a_2}{a_3}} + \bar{C} \quad (23)$$

With

$$C_{01} = a_1.C_0.A + a_2.A$$
$$C_{02} = [a_1.A.(C_0.t_c s + C_t) + a_2.A.t_c s]K^{\frac{-1}{a3}}$$
$$\bar{C} = a_1.C_o.(t_h + t_R) + a_2(t_h + t_R)$$

In equation (23) the last term is neglected for optimization calculations, since it is constant and therefore does not affect the calculation results.

$$Y = C_{01}.V^{-1}.f^{-1} + C_{02}.V^{\frac{1-a_3}{a_3}} f^{\frac{a_1-a_3}{a_3}}.d_c^{\frac{a_2}{a_3}} \quad (24)$$

After condensation by the method of Duffin [33] we find:

$$Y = C_k.V^{(\alpha w_2 - w_1)}.f^{(\beta w_2 - w_1)}.d_c^{\gamma w_2} \quad (25)$$

With:

$$C_k = \left(\frac{C_{01}}{w_1}\right)^{w_2} \cdot \left(\frac{C_{02}}{w_2}\right)^{w_2}$$
$$\alpha = \frac{1-a_3}{a_3}, \quad \beta = \frac{a_1 - a_3}{a_3} \quad \text{and} \quad \gamma = \frac{a_2}{a_3}$$
$$w_1 + w_2 = 1$$

w_1 and w_2 are the two weighting coefficients of the two terms in equation (25).

Using the logarithmic transformation, equation (25) becomes:

$$ln(Y) = ln(C_k) + (\alpha w_2 - w_1) ln(V) + (\beta w_2 - w_1) ln(f) + \gamma w_2 ln(d_c) \quad (26)$$
$$ln(Y) = C_{11} + C_1.x_1 + C_2.x_2 + C_3.x_3$$

With:

$$C_{11} = ln(C_k), \quad C_1 = \alpha w_2 - w_1$$
$$C_2 = \beta w_2 - w_1, \quad C_3 = \gamma w_2$$
$$x_1 = ln(V), \quad x_2 = ln(f), \quad x_3 = ln(d_c)$$

The general form of the function to optimize is:

$$min(x_0) = min(C_1.x_1 + C_2.x_2 + C_3.x_3)$$

The constant C_{11} is neglected since it remains constant regardless of the values of w_1 and w_2.

3 EXAMPLE OF APPLICATION

The objective function, as well as various constraints or limitations, make up the mathematical model of optimization. The parameters used for the numerical application are mentioned in Table 1.These conditions had been used by Agapiou [14].

Table 1: Machining parameters

Parameter	Value	Parameter	Value
L (mm)	203 mm	t_R *(mn/pass)*	0.13
D (mm)	152 mm	t_h *(mn/piece)*	1.5
V_{min} *(m/mn)*	30 m/min	θ_{max} *(C°)*	500
V_{max} *(m/mn)*	200 m/min	α_1	0.29
f_{min} *(mm/rev)*	0.254 mm/rev	α_2	0.35
f_{max} *(mm/rev)*	0.762 mm/rev	α_3	0.25
SF_{max} *(µm)*	2	*K*	193.3
SR_{max} *(µm)*	8	t_{cs} *(mn/edge)*	0.5
HP_{max} *(kW)*	5	C_o *($/min)*	0.1
F_{max} *(N)*	1100	C_t *($/edge)*	0.5

Source: Agapiou, J.S. (1992a).

We obtain the mathematical model in the form of the following equation system:

$$min(x_0) = min(C_1.x_1 + C_2.x_2 + C_3.x_3)$$

Under the constraints:

$$x_1 \leq 5.30\ ,\quad x_1 \geq 3.401$$
$$x_2 \leq 2.03\ ,\quad x_2 \geq 0.932$$
$$x_3 \leq 1.625\ ,\quad x_3 \geq 0.239$$

$$0.91x_1 + 0.78x_2 + 0.75x_3 \leq 6.694$$
$$-0.52x_1 + 1.004x_2 + 0.25x_3 \leq -5.21$$
$$0.4x_1 + 0.2x_2 + 0.105x_3 \leq 2.393$$
$$-0.1013x_1 + 0.725x_2 + 0.75x_3 \leq 1.934$$

The model in question compound of an objective function minimized and ten (10) constraints including three (3) inequalities greater or equal and seven (7) inequalities less than or equal.

The constants of the objective function have the following values:

$$C_1 = \alpha w_2 - w_1\ ,\quad \alpha = \frac{1-a_3}{a_3} = \frac{1-0.25}{0.25} = 3$$
$$C_2 = \beta w_2 - w_1\ ,\quad \beta = \frac{a_1 - a_3}{a_3} = \frac{0.29-0.25}{0.25} = 0.16$$
$$C_3 = \gamma w_2\ ,\quad \gamma = \frac{a_2}{a_3} = \frac{0.35}{0.25} = 1.4$$

We obtain accordingly:

$$C_1 = 3w_2 - w_1 \;\; , \;\; C_2 = 0.16w_2 - w_1 \;\; , \;\; C_3 = 1.4w_2$$

With:

$$w_1 + w_2 = 1 \;\; , \quad w_1 \text{ and } w_2 \geq 0$$

So the objective function takes the following form:

$$min(x_0) = min\left((3w_2 - w_1).x_1 + (0.16w_2 - w_1).x_2 + 1.4w_2.x_3\right)$$

To determine the suitable weighting coefficient values and the goal function's minimal value, we vary w_1 and w_2.

The expressions of the objective function for different values of the weighting coefficients are given in Table 2.

Table 2: Expression of the objective function

w_1	w_2	The objective function
9	0.1	$(2.6x_1 + 0.044x_2 + 1.26x_3)$
0.1	0.9	$(-0.6x_1 - 0.884x_2 + 0.14x_3)$
0.2	0.8	$(-0.2x_1 - 0.768x_2 + 0.28x_3)$
0.8	0.2	$(2.2x_1 - 0.072x_2 + 1.12x_3)$
0.3	0.7	$(0.2x_1 - 0.652x_2 + 0.42x_3)$
0.7	0.3	$(1.8x_1 - 0.188x_2 + 0.98x_3)$
0.4	0.6	$(0.6x_1 - 0.536x_2 + 0.56x_3)$
0.6	0.4	$(1.4x_1 - 0.304x_2 + 0.84x_3)$
0.5	0.5	$(x_1 - 0.42x_2 + 0.7x_3)$

Source: Authors.

The resolution of the optimization problem was made by the Simplex method in language Fortran 90.

The best solution, which gives the objective function with the smallest value, is obtained with the weighting coefficients w_1= 0.8 and w_2 = 0.2.

After determining the best combination of weighting coefficients, we use these coefficients for the calculation of the multi-criteria objective function.

After finding the values of the variables x_1, x_2, x_3 we go to the exponential form to find the values of the optimal cutting speeds:

$$V = e^{x_1} \quad , \quad f = \frac{e^{x_2}}{10} \quad , \quad d_c = e^{x_3}$$

The calculation results are given in Table 3.

Table 3: The optimized turning parameters

d_c(mm)	f (mm/rev)	V(m/min)	C_u($)	T_u(min)
1.27	0.76	135	0.382	2.668
2.54	0.636	122	0.525	3.09
3.81	0.554	118.72	0.688	3.440
5.08	0.461	110.5	0.795	3.934

Source: Authors.

Table 4 presents the values of the physical constraints for each operation.

Table 4: Values of physical constraints for each operation

d_c (mm)	f (mm/rev)	V (m/min)	F (N)	HP (KW)	SR (μm)	θ (°C)
0.50	0.761	172.3	244	1.94	2.73	499
0.75	0.761	154.4	335	2.38	4.88	499
1.00	0.761	142.6	419	2.75	5.91	498
1.25	0.761	134.2	498	3.07	6.86	497
1.50	0.761	129.1	573	3.40	7.6	499
1.75	0.735	126.2	629	3.64	7.9	499
2.00	0.700	124.9	672	3.84	7.89	499
2.50	0.645	122.7	750	4.19	7.87	499
3.00	0.603	120.9	820	4.49	7.85	499
4.00	0.539	117.7	940	4.99	7.82	498
5.00	0.466	111.0	1006	4.99	7.77	483

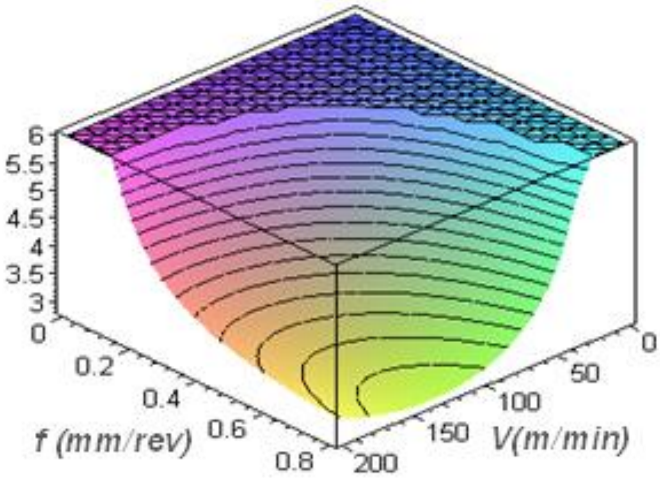
Source: Authors.

3.1 GRAPHIC REPRESENTATION

The two most important variables for minimizing the cost of production are the feed rate and the cutting speeds, *f* and *V*. By fixing the other variables at their optimal value, we can plot the variation of the multi-objective function as a function of these two variables.

The Figure 1 represent the graphical representation of the production time according to feed rate *f* and cutting speed *V* for a depth of cutting given equal to 2.54 mm [17, 19].

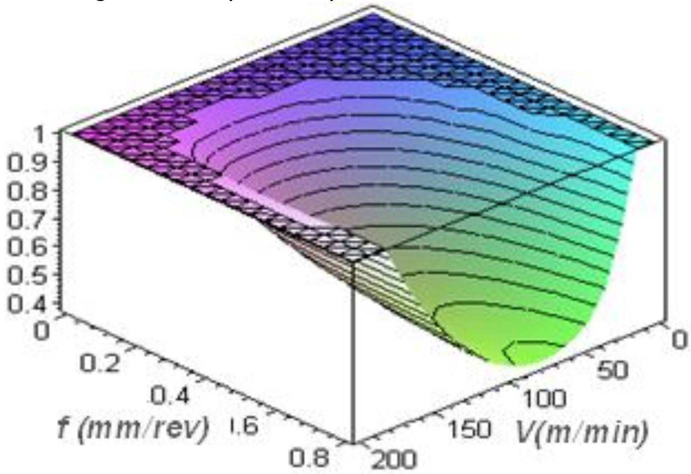
Figure 1: Graphical representation of production time.



Source: Authors.

The Figure 2 represents the function of the production cost according to feed rate f and cutting speed V for a depth of cutting given equal to 2.54 mm [17, 19].

Figure 2: Graphical representation of cost time



Source: Authors.

It can be seen in Figures 2 and 3 that the greater the advance and the lower the speed, the lower the cost. This is easily explained by the fact that the higher the feed rate, the shorter the machining time, which reduces the cost related to the machining time. As for the cutting speed, the lower it is, the longer the tool lasts, which reduces the costs related to the tool. The minimization of the C_u cost will be limited by the active constraints. The most important constraint regarding these variables is the cutting force.

4 DISCUSSION OF RESULTS

From the Table 3 it is noted that for a cutting depth of 2.54 mm we found a cutting speed *V* = 122 m/min and an advance *f* = 0.641 mm/rev; these two values respect all the constraints limitations and represent the optimal operating point.

According to the results of Table 4, we note that:

1- Machining with large cutting depths causes the increase cutting forces, power consumption and decreases tool life.

2- The temperature is close to the limit of the maximum permissible temperature θ_{max} = 500 ° C, for all operations which requires lubrication.

3- Machining with optimal speeds brings the physical constraints to their maximum allowed limits, so the optimal choice of cutting parameters is very important because:

- - Machining with too low cutting depths causes excessive heat, poor shavings control and high costs, but the machining with too deep cutting depths consumes too much power, causes rapid wear of cutting tools and requires too much cutting force.
- - Machining with low feed rates causes tool wear and high costs, but machining with too high feeds causes poor control and welding of the shavings, poor surface condition and too much power.

The smaller value of cutting speed results in a high production cost. It is due to the fact that a smaller cutting speed increases the production time of parts and the associated costs with it. Also, it will decrease the profit due to the production of a lesser number of parts. However, excessive tool wear and the increased downtime will result in a high production cost if the cutting speed is set too high. The optimum cutting speed is somewhere between “too slow” and “too fast” which will yield the minimum production cost.

It has been shown that geometric programming approach can also be applied effectively and efficiently to optimize the turning operation.

The present method is a generalized solution method so that it can be easily employed to consider the optimization models of turning regarding various objectives and constraints.

In the machining models, no specific workpiece and tool was identified. Therefore, the solution approach can be used with any workpiece for turning optimization problems.

5 CONCLUSION

This work presents of an optimization strategy to determine the optimum cutting parameters for turning processes. The maximum cutting speed, maximum feed rate, maximum available power, and surface roughness were all used as constraints. Our interest was related particularly to the model presented by Agapiou [14,15] which represents one of the rare approaches for the resolution of multi-objective optimization. According to the mathematical model, the proposed method provides a systematic and efficient methodology for obtaining the lowest production cost and time for turning. It has been demonstrated that the geometric programming method may be successfully used to reduce the cost and time of the turning process.

The results obtained demonstrated the advantages of the methodology used in modeling by presenting the interest of mastering the mathematical tool in the resolution of problems arising from the field of industry. Thus, they presented the capacity of the algorithms used (geometric programming) in solving nonlinear optimization problems with constraints.

NOMENCLATURE

Symbol	Description
a_1, a_2	Empirical constants for tool life equation.
C_i	machine idle cost due to loading and unloading operations and tool idle motion time ($/piece)
C_m	Cutting cost by actual time in machining ($/piece)
C_r	Tool replacement cost ($/piece)
C_t	Tool cost ($/piece)
C_u	Production cost ($/piece)
d_c	Depth of material to be removed (mm)
d_r , d_f	depth of cut for each pass for rough and finish machining (mm)
f_r, f_f	Feed rates for rough and finish machining (mm/rev)
D	Diameter of workpiece (mm)
F	Cutting force (N)
HP	Machine horsepower (kW)
K	Tool life constant.
L	Work piece length to be machined (mm)
SF	Surface finish roughness (µm)
SR	Surface roughness (µm)
t_{cs}	Tool change time (min./edge)

t_h	Auxiliary time (min)
t_m	Cutting time (min)
t_R	Return time (min)
T_u	Production time (min)
V_r, V_f	Cutting speeds in rough and finish machining (m/min)
w_1, w_2	The weighting coefficients
θ	Average cutting temperature C°

REFERENCES

[1] BELLOUFI, A.; ASSAS, M.; REZGUI I. Optimization of Turning Operations by Using a Hybrid Genetic Algorithm with Sequential Quadratic Programming. **Journal of Applied Research and Technology**, v. 11, pp. 88-94, 2013. DOI: 10.1016/S1665-6423(13)71517-7

[2] EDGEWORTH, F. Y. **Mathematical psychics**. London: Kegan Paul, 1881.

[3] PARETO, V. **Manuale di Economia Politica Societa Editrice Libraria**, Milano, 1906. English Translation: Pareto, V.: Manual of Political Economy, translated by Schwier, A. S.. Augustus 1170 M.Kelley Publishers, New York, 1971.

[4] EICHFELDER, G.; JAHN, J. () Vector and set optimization. *In*: GRECO, S. (Ed.), **Multiple Criteria Decision Analysis**. State of the Art Surveys, Springer, Heidelberg, 2016. p. 695-737.

[5] WANG, Y. C. (2007). A note on optimization of multipass turning operations using ant colony system. **International Journal of Machine Tools and Manufacture**, v. 47, n. 12-13, p. 2057-2059. DOI: 10.1016/j.ijmachtools.2007.03.001

[6] SHUTONG, X.; YINBIAO, G. Intelligent selection of machining parameters in multi-pass turnings using a GA-based approach. **Journal of Computer Information Systems**, v. 5, p. 1714-1721, 2011. DOI: 10.1155/2014/592627

[7] CROOKALL, J. R.; VENKATARAMANI, N. Computer optimization of multipass turning. **International Journal of Production Research**, v. 9, n. 2, p. 247–259, 1971. DOI: 10.1080/00207547108929876

[8] KALS, H. J. J. (1977). Computer aid in the optimization of turning conditions in multi-cut operations. **Proceedings of CIRP Conference**, p. 465–471.

[9] LAMBERT, P. K.; WALVEKAR, A. G. Optimization of multipass machining operations. **International Journal of Production Research**, v. 16, n. 4, p. 259–265, 1978. DOI: 10.1080/00207547808930018

[10] ÖZ, E.; GÜZEL, N.; ALP, S. An alternative approach to the solution of multi-objective geometric programming problems. **Open Journal of Optimization**, v. 6, p. 11-25, 1961. DOI: 10.4236/ojop.2017.61002

[11] ÖZ, Ersoy; GÜZEL, Nuran; ALP, Selçuk. An Alternative Approach to the Solution of Multi-Objective Geometric Programming Problems. **Open Journal of Optimization**, v. 6, n. 1, p. 11-25, 2017. DOI: 10.4236/ojop.2017.61002

[12] TOKSARI, M. D. () Taylor Series Approach to Fuzzy Multiobjective Linear Fractional Programming. **Information Sciences**, v. 178, p. 1189-1204, 2008. DOI: 10.1016/j.ins.2007.06.010

[13] GÜZEL, N.; SIVRI, M. () Taylor Series Solution of Multi Objective Linear Fractional Programming Problem. **Trakya University Journal of Science**, v. 6, p. 91-98, 2005.

[14] AGAPIOU, J. S. The optimization of machining operations based on a combined criterion, Part 2: Multipass operations. **Journal of Engineering for Industry**, v. 114, p. 508-513, 1992a. DOI: 10.1115/1.2900705

[15] AGAPIOU, J. S. The optimization of machining operations based on a combined criterion, Part 1: The use of combined objectives in single-pass operations. **Journal of Engineering for Industry**, v. 114, n. 4, p. 500-507, 1992b. DOI: 10.1115/1.2900704

[16] WANG, J.; KURIYAGAWA, T.; WEI, X. P.; GOU, G. M. Optimization of cutting conditions using a deterministic approach. **International Journal of Machine Tools & Manufacture**, v. 42, 1023-1033, 2002. DOI: 10.1016/S0890-6955(02)00037-8

[17] DJENANE, M.; DJARI, D.; BENBOUTA, R.; ASSAS, M. Multi pass optimization of cutting conditions by using the genetic algorithms. **Research Journal of Applied Sciences, Engineering and Technology**, v. 13, n. 3, p. 223-231, 2016. DOI: 10.19026/rjaset.13.2934

[18] SOFUOĞLU, M. A.; ARAPOĞLU, R. A.; ORAK S. Multi objective optimization of turning operation using hybrid decision making analysis. **Anadolu University Journal of Science and Technology A- Applied Sciences and Engineering**, v. 18, n. 3, p. 595-610, 2017. DOI:10.18038/aubtda.287801

[19] DJARI, D.; ASSAS, M.; DJENANE, M. Optimization of the Conditions of Machining Based on a Criterion Combined by Genetic Algorithms. **IRENA**, v. 3, n. 4, p. 100-104, 2015.

[20] AREZOO, B.; RIDGWAY, K.; AL-AHMARI, A. M. Selection of cutting tools and conditions of machining operations using an expert system. **Computers in Industry**, v. 42, p. 43-58, 2000. DOI: 10.1016/S0166-3615(99)00051-2

[21] CHOUDHURY, S. K.; APPA RAO, I. V. K. Optimization of cutting parameters for maximizing tool life. International Journal of Machine Tools & Manufacture, v. 39, p. 343-353, 1999. DOI: 10.1016/ S0890-6955(98)00028-5

[22] ZUPERL, U.; CUS, F. Optimization of cutting conditions during cutting by using neural networks. **Robotics and Computer Integrated Manufacturing**, v. 19, p. 189-199, 2003. DOI:10.1016/S0736-5845(02)00079-0

[23] DU, M.; CHENG, Z.; ZHANG, Y.; WANG, S. Multiobjective optimization of tool geometric parameters using genetic algorithm. **Complexity,** p. 1-24, 2018. DOI: 10.1155/2018/9692764

[24] MIODRAGOVIC, G. R.; DORDEVIC, V.; BULATOVIC, R.; PETROVIC, A. Optimization of multi-pass turning and multi-pass face milling using subpopulation firefly algorithm. **Journal of Mechanical Engineering Science**, v. 233, p. 1520–1540, 2019. DOI: 10.1177/0954406218774378

[25] AMIOLEMHEN, P.; ESEIGBE, J. Genetic algorithms solution to the single - objective machining process optimization time model. **Journal of Mechanical and Energy Engineering**, v. 3, n. 41, p. 13-24, 2019. DOI: 10.30464/jmee.2019.31.13

[26] SAI, N. K.; CHARYULU, T. N.; SIVA NAYAK, M. Multi objective optimization of machining parameters by using weighted sum genetic algorithm approaches. **International journal of engineering research and technology**, v. 1, n. 7, p. 1-7, 2012.

[27] ARMAREGO, E. J. A.; BROWN, R. H. **The machining of metals**. Prentice-Hall, 1969.

[28] LIBAO, A. **Optimization of machining parameters in multi-pass turning and milling operations**. Master (Dissertation) – Concordia University, 2003.

[29] OJHA, A. K.; BISWAL, K. K. Multi-objective Geometric Programming Problem with Weighted Mean Method. **International Journal of Computer Science and Information Security**, v. 7, n. 2, p. 82-86, 2010. DOI:10.48550/arXiv.1003.1477

[30] LIU, S. T. Posynomial Geometric Programming with parametric uncertainty. **Europian Journal of Operation Research**, v. 168, p. 345-353, 2006. DOI: 10.1016/j.ejor.2004.04.046

[31] JHA, N. K. Optimizing the number of tools and cutting parameters in multi-tool turning for multiple objectives through geometric programming. **Applied Mathematical Modeling**, v. 10, p. 162-169, 1986. DOI: 10.1016/0307-904x(86)90041-7

[32] DUFFIN, R. J.; PETERSON, E.L.; ZENER, C. **Geometric programming**: theory and application. New York: John Wiley and Sons, 1967.

[33] DUFFIN, R. J. Linearizing geometric program. **SIAM Review**, v. 12, n. 2, 1970211-227, DOI: 10.1137/1012043