



Solution of nonlinear partial differential
Equations by adomian decomposition method

Solução de diferencial parcial não-linear
Equações pelo método de decomposição adomiana

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Abstract—In this paper we introduce a relatively new method, based on Adomian decomposition method (ADM), for solving nonlinear partial differential equations (NPDE). This method provides the solution as an infinite series in which each term can be easily determined. We shall consider some particular examples to discuss the accuracy and convergence of the method. The results obtained by this way have been compared with the exact solution to show the efficiency of the method.

Keyword— Decomposition method, Adomian polynomials, Nonlinear differential equations.

1. INTRODUCTION

Many problems in physics-mathematics, theoretical chemistry, biology, mathematical finance and engineering in general are modeled by boundary value problems or initial value problems involving NPDE. Nonlinear differential equations are also very useful for modeling such diverse application problems as: incompressible fluid waves, magnetic waves in cold plasma, acoustic waves in an anharmonic crystal, investment options in an illiquid or non-constant volatility market and so on. However, the equations governing the model are difficult to solve analytically and sometimes it is impossible to find analytical solutions, so we must try to solve them with numerical methods such as the Galerkin method, finite differences, finite element, etcetera. However, most of the methods developed so far in mathematics and their applications are used to solve linear differential equations. The decomposition method developed by mathematician and engineer George Adomian (1923-1996), has been very useful in applied mathematics in general [1], [2]. The Adomian decomposition method (ADM) has the advantage that it converges to the exact solution in a large majority of very important cases in applications and can be handled easily for a wide class of differential equations (ordinary and partial) both linear and nonlinear. The application of the method has proven to yield reliable results and only a few terms are needed to obtain, in some cases, the exact solution or to find an approximate solution to a reasonable degree of accuracy in real physical models. Moreover, the method does not require linearization or perturbation of any kind to work effectively.

In the present work we will make use of ADM to solve specifically nonlinear partial differential equations, although in some of the examples in section 3 we will apply it to a linear case so that the reader can see that the method is also efficient in those situations.

The paper is organized as follows: In section 2 we present the Adomian decomposition method in general for NPDE. In section 3 we will apply AMD to specific examples of nonlinear differential equations and we will give a particular example in which the nonlinear term is null in order



to illustrate the possible application of ADM also to NPDE. Finally, section 4 is reserved for conclusions.

2. BRIEF PRESENTATION OF ADM TO SOLVE NPDE

The Adomian Decomposition Method (ADM) allows finding an analytical solution in the form of a series and consists of identifying in the given equation the linear and nonlinear parts, to then invert the differential operator of higher order that is in the linear part and then consider the function to be known as a series whose summands, by the present method will be well determined, then the nonlinear part is decomposed in terms of the Adomian polynomials. We define the initial and/or boundary conditions and the terms involving the independent variable as initial approximation. Then, we find successively the terms of the series that gives us the solution of the problem establishing a recursive relation [5, 6].

In general, the method to follow is as follows: given an ordinary or partial differential equation:

$$(1) \quad Fu(x, t) = g(x, t)$$

with initial condition

$$(2) \quad u(x, 0) = f(x)$$

where F represents a differential operator (in general, nonlinear) that involves both linear and nonlinear terms and then the equation (1) can be written as

$$(3) \quad L_t u(x, t) + Ru(x, t) + Nu(x, t) = g(x, t)$$

where $L_t = \frac{\partial}{\partial t}$, R is a linear operator involving partial derivatives with respect to x and N is a nonlinear operator; g is a non-homogeneous term independent of u .

Operating algebraically $L_t u(x, t)$,

$$(4) \quad L_t u(x, t) = g(x, t) - Ru(x, t) - Nu(x, t)$$

Since L is invertible, operating on (4) with inverse $L_t^{-1}(\cdot) = \int_0^t (\cdot) dt$ we obtain

$$(5) \quad L_t^{-1} L_t u(x, t) = L_t^{-1} g(x, t) - L_t^{-1} Ru(x, t) - L_t^{-1} Nu(x, t)$$

then, an equivalent expression to (5) is

$$(6) \quad u(x, t) = f(x) + L_t^{-1} g(x, t) - L_t^{-1} Ru(x, t) - L_t^{-1} Nu(x, t)$$

where $f(x)$ is the constant of integration (with respect to t) satisfying $L_t f = 0$. For problems with initial value at $t = t_0$, we have conveniently defined L^{-1} for $L = \frac{\partial^n}{\partial x^n}$ as the n -times iterated defined from t_0 to t .

ADM assumes the solution of (1), (2) in series form for the unknown function $u(x, t)$ given by,

$$(7) \quad u(x, t) = \sum_{n=0}^{\infty} u_n(x, t)$$

The nonlinear term $Nu(x, t)$ by means of ADM decomposes as

$$(8) \quad Nu(x, t) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \dots, u_n)$$

where the sequence $\{A_n\}_{n=0}^{\infty}$, is the so called Adomian polynomial sequence, for the calculation of the Adomian polynomials given in the formula (9) a good reference is [3]. In [3] we can see that the explicit calculation of each of the A_n is really simple and one has: $A_0(u_0) = N(u_0)$

$$A_1(u_0, u_1) = N'(u_0)u_1$$

$$A_2(u_0, u_1, u_2) = N'(u_0)u_2 + \frac{u_1^2}{2!} N''(u_0)$$



$$A_3(u_0, u_1, u_2, u_3) = N'(u_0)u_3 + N''(u_0)u_1u_2 + \frac{u_1^3}{3!}N'''(u_0)$$

$$A_3(u_0, \dots, u_4) = u_4N'(u_0) + (\frac{1}{2!}u_2^2 + u_1u_3)N''(u_0) + \frac{u_1^2u_2}{2!}N'''(u_0) + \frac{u_1^4}{4!}N^{(iv)}(u_0)$$

$$\vdots$$

Therefore, its calculation can be summarized as follows

$$(9) \quad A_n(u_0, u_1, \dots, u_n) = \frac{1}{n!} \frac{d^n}{d\alpha^n} [N(\sum_{k=0}^n \alpha^k u_k)]|_{\alpha=0}.$$

Now, substituting (7), (8) and (9) in equation (6) we obtain

$$(10) \quad \sum_{n=0}^{\infty} u_n(x, t) = f(x) + L_t^{-1}g(x, t) - L_t^{-1}R \sum_{n=0}^{\infty} u_n(x, t) - L_t^{-1} \sum_{n=0}^{\infty} A_n(u_0, u_1, \dots, u_n).$$

By identifying u_0 as $f(x) + L_t^{-1}g(x, t)$ en (10), we can write

$$\begin{aligned} u_0(x, t) &= f(x) + L_t^{-1}g(x, t), \\ u_1(x, t) &= -L_t^{-1}Ru_0(x, t) - L_t^{-1}A_0(u_0), \\ &\vdots \\ u_{n+1}(x, t) &= -L_t^{-1}Ru_n(x, t) - L_t^{-1}A_n(u_0, \dots, u_n). \end{aligned}$$

From which we can deduce the following recursive algorithm [13]:

$$(11) \quad \begin{cases} u_0(x, t) = f(x) + L_t^{-1}g(x, t), \\ u_{n+1}(x, t) = L_t^{-1}Ru_n(x, t) - L_t^{-1}A_n(u_0, u_1, \dots, u_n), \quad n = 0, 1, 2, \dots \end{cases}$$

With the recursive algorithm established in the equation (11) we can have an approximation to the solution of (1), (2) by means of the series

$$(12) \quad u_k(x, t) = \sum_{n=0}^k u_n(x, t), \quad \text{where} \quad \lim_{k \rightarrow \infty} \sum_{n=0}^k u_n(x, t) = u(x, t).$$

The decomposition of the series solution generally converges very quickly. The rapidity of this convergence means that few terms are needed for the analysis of the solution. The conditions for which the method converges have been studied mainly in the references [7], [8], [9] and [10].

3. EXAMPLES OF APPLICATION

Next we will solve some partial differential equations using the method described in the previous section.

Example 1

As a first example, we will take the following initial value problem

$$(13) \quad \begin{cases} \frac{\partial u}{\partial t} = \frac{1}{2} \frac{\partial^2 u}{\partial r^2} + e^{-u} + \frac{1}{2} e^{-2u}, \\ u(r, 0) = \ln(r + 2). \end{cases}$$

Let's consider the nonlinear term N as

$$Nu = e^{-u} + \frac{1}{2} e^{-2u},$$



and applying the formula for calculating polynomials, i.e., using

$$A_n(u_0, u_1, \dots, u_n) = \frac{1}{n!} \frac{d^n}{d\alpha^n} [N(\sum_{k=0}^n \alpha^k u_k)]|_{\alpha=0} \quad n \geq 0$$

we have:

$$\begin{aligned} A_0(u_0) &= N(u_0) = e^{-u_0} + \frac{1}{2}e^{-2u_0} \\ A_1(u_0, u_1) &= N'(u_0)u_1 = -u_1e^{-u_0} - u_1e^{-2u_0} \\ A_2(u_0, u_1, u_2) &= N''(u_0)\frac{u_1^2}{2!} + N'(u_0)u_2 \\ &= \frac{u_1^2}{2}(e^{-u_0} + 2e^{-2u_0}) + u_2(-e^{-u_0} - e^{-2u_0}) \\ A_3(u_0, u_1, u_2, u_3) &= N'''(u_0)\frac{u_1^3}{3!} + N''(u_0)u_1u_2 + N'(u_0)u_3 \\ &= \frac{u_1^3}{6}(-e^{-u_0} - 4e^{-2u_0}) + u_1u_2(e^{-u_0} + 2e^{-2u_0}) - u_3(e^{-u_0} + e^{-2u_0}) \\ &\vdots \end{aligned}$$

Calculating the partial sums of the Adomian series:

$$\begin{aligned} S_0 &= u_0 = \ln(r + 2) \\ S_1 &= u_0 + u_1 = \ln(r + 2) + \frac{t}{r + 2} \\ S_2 &= u_0 + u_1 + u_2 = \ln(r + 2) + \frac{t}{r + 2} - \frac{t^2}{2(r + 2)^2} \\ S_3 &= u_0 + u_1 + u_2 + u_3 = \ln(r + 2) + \frac{t}{r + 2} - \frac{t^2}{2(r + 2)^2} + \frac{t^3}{3(r + 2)^3} \\ &\vdots \\ S_m &= u_0 + u_1 + \dots + u_m = \ln(r + 2) + \frac{t}{r + 2} - \frac{t^2}{2(r + 2)^2} + \dots + \frac{(-1)^{m+1}t^m}{m(r + 2)^m} \end{aligned}$$

and so we note that

$$(14) \quad u(r, \tau) = \ln(r + 2) + \frac{t}{r + 2} - \frac{t^2}{2(r + 2)^2} + \dots + \frac{(-1)^{m+1}t^m}{m(r + 2)^m} + \dots$$

by taking the sum of the first terms, we can estimate that the series converges to $\ln(\frac{t+r+2}{r+2})$. Then, using (14), we have

$$(15) \quad u(r, t) = \ln(r + 2) + \ln(\frac{t + r + 2}{r + 2}) = \ln(r + t + 2).$$

The reader can easily verify by calculating the corresponding partial derivatives that (15) is a solution of the model given by the NPDE (13). The NPDE (13) has served as a mathematical model to describe the growth of a brain tumor (glioblastomas) under medical treatment [12].

Example 2

Consider the following NPDE type nonlinear heat equation given by the following initial value problem

$$(16) \quad \begin{cases} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 2u^3, \\ u(x, 0) = \frac{2x+1}{x^2+x+1}. \end{cases}$$



In this case, the first Adomian polynomials are

$$\begin{aligned} A_0 &= -2u_0^3 \\ A_1 &= -6u_0^2u_1 \\ A_2 &= -6u_0u_1^2 - 6u_0^2u_2 \end{aligned}$$

and with them we get

$$\begin{aligned} u_0(x, t) &= \frac{1 + 2x}{x^2 + x + 1} \\ u_1(x, t) &= L_t^{-1}L_{xx}(u_0) - 2L_t^{-1}(u_0^3) = \frac{-6(1 + 2x)}{(x^2 + x + 1)^2}t \\ u_2(x, t) &= L_t^{-1}L_{xx}(u_1) - 6L_t^{-1}(u_0^2u_1) = \frac{36(1 + 2x)}{(x^2 + x + 1)^3}t^2 \\ u_3(x, t) &= L_t^{-1}L_{xx}(u_2) - 6L_t^{-1}(u_0u_1^2 + u_0^2u_2) = \frac{-216(1 + 2x)}{(x^2 + x + 1)^4}t^3 \\ &\vdots \end{aligned}$$

substituting in the equation (12) we obtain

$$u(x, t) = u_0(x, t) + u_1(x, t) + u_2(x, t) + u_3(x, t) + \dots$$

from which we obtain the approximation to the solution

$$(17) \quad u(x, t) \approx \frac{1 + 2x}{x^2 + x + 1} - \frac{6(1 + 2x)}{(x^2 + x + 1)^2}t + \frac{36(1 + 2x)}{(x^2 + x + 1)^3}t^2 - \frac{216(1 + 2x)}{(x^2 + x + 1)^4}t^3.$$

The reader who wants to see how good this approximation is can compare (17) with the exact solution of (16), which is

$$u(x, t) = \frac{1 + 2x}{x^2 + x + 6t + 1}.$$

In the following example we will consider a linear partial differential equation to illustrate that in the case in which the nonlinear term is null, that is, $N = 0$ in the equation (3); the method described in the previous section also works efficiently. Furthermore, in the following example we will consider the modification of the method made in [4], which consists in decomposing the initial condition into two or more algebraic summands.

Example 3

Consider the following linear partial differential equation given by

$$(18) \quad \begin{cases} u_{tt} + u_{xx} + u = 0, \\ u(x, 0) = 1 + \text{sen}x, \quad u_t(x, 0) = 0. \end{cases}$$

In the present example, the nonlinear term is null, i.e., $N(x, t) = 0$, therefore $A_n = 0$ for each $n \geq 0$. In addition, we will consider the invertible operator $L_{tt} = \frac{\partial^2}{\partial t^2}$, so

$$L_{tt}^{-1}(\cdot) = \int_0^t \int_0^t (\cdot) ds dw.$$

Solving the equation (18) for u_{tt} and applying L_{tt}^{-1} we obtain

$$u(x, t) = 1 + \text{sen}x - L_{tt}^{-1}(u + u_{xx}).$$

Now we are going to take into account the decomposition proposed in [4], that is, we are going to decompose f as $f(x) = f_0(x) + f_1(x)$. In this decomposition, $f_0(x) = 1$ and $f_1(x) = \text{sen}x$.



Calculating:

$$\begin{aligned} u_0(x, t) &= 1, \\ u_1(x, t) &= \text{sen}x - L_{tt}^{-1}(u_0 + u_{0,xx}) = \text{sen}x - \frac{1}{2}t^2, \\ u_2(x, t) &= -L_{tt}^{-1}(u_1 + u_{1,xx}) = \frac{1}{4!}t^4, \\ u_3(x, t) &= -L_{tt}^{-1}(u_2 + u_{2,xx}) = -\frac{1}{6!}t^6, \\ u_4(x, t) &= -L_{tt}^{-1}(u_3 + u_{3,xx}) = \frac{1}{8!}t^8, \end{aligned}$$

from which it can be deduced that in general that,

$$u_k(x, t) = -L_{tt}^{-1}(u_{k-1} + u_{k-1,xx}) = \frac{(-1)^k}{(2k)!}t^{2k}, \quad k \geq 2.$$

Therefore the solution of (18) in series is

$$u(x, t) = \text{sen}x + 1 - \frac{1}{2!}t^2 + \frac{1}{4!}t^4 - \frac{1}{6!}t^6 + \dots \pm \frac{1}{(2k)!}t^{2k} + \dots,$$

identifying the Taylor series of $f(t) = \text{cost}$ around the zero, we have that the solution of (18) is

$$(19) \quad u(x, t) = \text{sen}x + \text{cost}.$$

The reader can verify that u given in equation (19) is the exact solution of (18).

Example 4

Consider the following nonlinear partial equation Korteweg-de Vries (KdV) type given in the initial value problem:

$$(20) \quad \begin{cases} u_t + u_x + u^2u_{xx} + u_xu_{xx} - 20u^2u_{xxx} + u_{xxxxx} = 0, \\ u(x, 0) = \frac{1}{x}. \end{cases}$$

We are going to use ADM combined with Laplace transform \mathcal{L} as set in [11].

We will take $u_0(x, t) = \frac{1}{x}$; solving $L_t u$ from (20) and taking \mathcal{L} , we have

$$(21) \quad Su(x, S) - u(x, 0) = \mathcal{L}\{20u^2u_{xxx} - u_{xxxxx} - u_x - u^2u_{xx} - u_xu_{xx}\}$$

Applying the initial condition we obtain

$$(22) \quad u(x, S) = \frac{1}{Sx} + \frac{1}{S}\mathcal{L}\{20u^2u_{xxx} - u_{xxxxx} - u_x - u^2u_{xx} - u_xu_{xx}\}.$$

By taking \mathcal{L}^{-1} in (22) we get

$$(23) \quad u(x, t) = \frac{1}{x} + \mathcal{L}^{-1}\left[\frac{1}{S}\mathcal{L}\{20u^2u_{xxx} - u_{xxxxx} - u_x - u^2u_{xx} - u_xu_{xx}\}\right],$$

for the nonlinear parts; through the Adomian polynomials

$$A_n = u^2u_{xx}, \quad B_n = u_xu_{xx}, \quad C_n = u^2u_{xxx}$$

from where

$$\begin{aligned} A_0 &= u_0^2u_{0xx}, & A_1 &= 2u_0u_1u_{0xx} + u_0^2u_{1xx} \\ A_2 &= 2u_0u_2u_{0xx} + u_1^2u_{0xx} + 2u_0u_1u_{1xx} + u_0^2u_{2xx} \\ A_3 &= 2u_0u_3u_{0xx} + 2u_1u_2u_{0xx} + u_1^2u_{1xx} + 2u_0u_2u_{1xx} + 2u_0u_1u_{2xx} + u_0^2u_{3xx} \\ &\vdots \end{aligned}$$



$$\begin{aligned}
 B_0 &= u_{0x}u_{0xx}, & B_1 &= u_{0x}u_{1xx} + u_{0xx}u_{1x} \\
 B_2 &= u_{2x}u_{0xx} + u_{1x}u_{1xx} + u_{0x}u_{2xx} \\
 B_3 &= u_{0xx}u_{3x} + u_{1xx}u_{2x} + u_{1x}u_{2xx} + u_{0x}u_{3xx} \\
 &\vdots
 \end{aligned}$$

$$\begin{aligned}
 C_0 &= u_0^2u_{0xxx}, & C_1 &= 2u_0u_1u_{0xxx} + u_0^2u_{1xxx} \\
 C_2 &= 2u_0u_2u_{0xxx} + u_1^2u_{0xxx} + 2u_0u_1u_{1xxx} + u_0^2u_{2xxx} \\
 C_3 &= 2u_0u_3u_{0xxx} + 2u_1u_2u_{0xxx} + u_1^2u_{1xx} + 2u_0u_2u_{1xxx} \\
 &\quad + 2u_0u_1u_{2xxx} + u_0^2u_{3xxx} \\
 &\vdots
 \end{aligned}$$

The equation (23) can be written as:

$$\begin{aligned}
 (24) \quad u(x, t) &= \sum_{n=0}^{\infty} u_n(x, t) \\
 &= \frac{1}{x} + \mathcal{L}^{-1} \left(\frac{1}{S} \mathcal{L} \left\{ 20C_n - A_n - B_n - \sum_{n=0}^{\infty} u_{nx} - \sum_{n=0}^{\infty} u_{nxxxx} \right\} \right),
 \end{aligned}$$

through ADM we obtain the recursive relation

$$(25) \quad \begin{cases} u_0(x, t) = \frac{1}{x}, \\ u_{k+1}(x, t) = \mathcal{L}^{-1} \left(\frac{1}{S} \mathcal{L} \{ 20C_k - A_k - B_k - u_{kx} - u_{kxxxx} \} \right), \quad k \geq 0. \end{cases}$$

Now, using the recursive relation (25) we obtain the sequence of the $\{u_n\}_{n \geq 0}$, which is given for the first terms by

$$\begin{aligned}
 u_0(x, t) &= \frac{1}{x}, \\
 u_1(x, t) &= \mathcal{L}^{-1} \left(\frac{1}{S} \mathcal{L} \{ 20C_0 - A_0 - B_0 - u_{0x} - u_{0xxxx} \} \right) = \frac{t}{x^2}, \\
 u_2(x, t) &= \mathcal{L}^{-1} \left(\frac{1}{S} \mathcal{L} \{ 20C_1 - A_1 - B_1 - u_{1x} - u_{1xxxx} \} \right) = \frac{t^2}{x^3}, \\
 u_3(x, t) &= \mathcal{L}^{-1} \left(\frac{1}{S} \mathcal{L} \{ 20C_2 - A_2 - B_2 - u_{2x} - u_{2xxxx} \} \right) = \frac{t^3}{x^4}, \\
 u_4(x, t) &= \mathcal{L}^{-1} \left(\frac{1}{S} \mathcal{L} \{ 20C_3 - A_3 - B_3 - u_{3x} - u_{3xxxx} \} \right) = \frac{t^4}{x^5}, \\
 &\vdots && \vdots && \vdots
 \end{aligned}$$

Then, using (7), the fifth-order nonlinear KdV equation is solved:

$$\begin{aligned}
 u(x, t) &= \sum_{n=0}^{\infty} u_n(x, t) \\
 &= \frac{1}{x} + \frac{t}{x^2} + \frac{t^2}{x^3} + \frac{t^3}{x^4} + \frac{t^4}{x^5} + \dots \\
 &= \sum_{n=0}^{\infty} \frac{t^n}{x^{n+1}}
 \end{aligned}$$



from which we have

$$(26) \quad u(x, t) = \frac{1}{x - t}.$$

The reader can easily verify that u given in equation (26) is the exact solution of (20).

4. CONCLUSIONES

In this paper, the Adomian ADM decomposition method has been explained and applied to solve NPDE, emphasizing that the method is also applicable to solve linear partial differential equations. In addition, the use of the method is illustrated through simple examples so that the reader can get to know it and use it in his work with differential equations and it is noted that it is possible to combine the method with the Laplace transform and the serial decomposition of the initial condition proposed in [4]. It is also noted that ADM is a simple method that does not require a large amount of calculations nor the linearization of the differential equation for its application.

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